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Division de l'Analyse Structurelle

Working Paper
77-03-01

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Kirk E. Hamilton
Structural Analysis Division
March 1977

FOREWORD

This paper has been written with several goals in mind, which accounts in part for its length and considerable variation in degree of technicality. The initial aim is to provide an analysis of the impact on the Canadian energy resource base of external trade in 1966, with all the richness of detail which an energy accounting model can provide. Given that this topic is likely to be of general interest, as an aid to understanding the study the second goal is to provide a succinct introduction to the methodology and terminology of energy analysis. And finally, the paper represents an addition to the theory of energy analysis in its chapters dealing with the assignment of energy requirements to goods and services in an economy which is open to external trade.

The study is so structured that the first three chapters give the reader an overview of the means and the results of the analysis, Chapter 1 providing the motivation for the study, Chapter 2 the methodology of Input-Output based energy analysis, and Chapter 3 a summary of the detailed calculation in subsequent sections. Chapter 4 addresses the manner in which imports affect the energy required to produce commodities in Canada. The fifth chapter measures the energy effects of external trade in total, breaking these down into two parts: direct trade in energy commodities, and trade in all other commodities. The remaining chapters may be viewed as a progressively finer decomposition of the total result into its

PREFACE

This paper has other objectives more general than its title, which accounts in part for its length and its considerable theoretical "economic" or "theoretical". The initial aim is to provide an analysis of the impact on the Canadian energy industry of the external shock in 1973, with all the increase of energy which [illegible] energy accounting works can provide. Given that this topic is likely to be of general interest, as an aid to those who want to study the subject good [illegible] provide a somewhat introduction to the methodology and terminology of energy analysis, and finally, the paper represents an addition to the theory of energy analysis in its treatment dealing with the assignment of energy requirements to goods and services in an economy which is open to external trade.

The study is set organized that the first three chapters give the reader an overview of the scope and the limits of the analysis, Chapter 1 providing the motivation for the study, Chapter 2 the methodology of input-output based energy analysis and Chapter 3 a summary of the detailed information in subsequent sections. Chapter 4 documents the impact of imports and exports on energy requirements of goods consumed in Canada. The fifth chapter analyses the energy effects of external trade in Canada, [illegible] between indirect trade in embodied energy and direct trade in energy commodities, and [illegible] its other consequences. The concluding chapter sums up [illegible] the comparatively large [illegible] of the [illegible] [illegible]

elementary components. This process begins in Chapter 6 through a comparison of the energy requirements per dollar of the total bill of goods exported and imported. Chapter 7 introduces the concept of a balanced trade calculation of energy requirements for commodities. This calculation is, by the arguments presented, perhaps the optimal measure of energy requirements in an open economy; it allows in addition a pure measure of the complex interaction between domestic demand and external trade, a capability which is explored in some detail. The eighth chapter examines in depth the bill of goods exported and how variations in this bill of goods affect balanced trade calculations. And finally, Chapter 9 gives mathematical proofs of assertions in the text.

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1. Introduction

Traditional economic theory and analysis with regard to external trade have been primarily concerned with the "comparative advantage" of economic regions, which leads to mutually beneficial trading, and the concomitant problem of the balance of payments between regions which this trade entails. Economic analysis of trade has attempted to measure the factor content of imports and exports, the two prime factors considered being labour and capital; for good reasons, particularly the ability of money to co-measure disparate commodities, this analysis has tended to be in terms of monetary value. This paper is concerned with Canadian trade in a specific class of commodities, the energy commodities, and, at a different level, the energy required to produce goods which are exported as well as that which is not required when goods are imported; moreover, physical units of measure are used in this study because these are more appropriate in dealing with goods which, at least in their traditional forms, have a finite physical extent.

The recent upsurge of interest in energy has a variety of causes, several of which impinge on external trade. The trade embargo and subsequent dramatic increase in petroleum prices in 1973 (by the OPEC countries) has led first of all to petroleum becoming a large factor in the balance of payments problems of importing nations, and secondly has made security of supply an important variable in the energy trade of these nations, Canada included. But security of supply considerations generally imply

increasing reliance on domestic energy sources to meet demand, and in Canada this has led to the hard realization of the finiteness of our energy resources. In fact one possible scenario for the future of the developed countries, characterized as they are by high rates of consumption, is that resource constraints, at least in the form of shortfalls of resources which are capable of profitable exploitation with current technology, will necessitate economic dislocations in the growth path of these nations, forcing major substitutions both of products and technologies. It seems likely that energy in its traditional forms, the fossil fuels, could be the first such resource constraint to be encountered.

Viewed in this manner, what is popularly termed "the energy crisis" is seen to be a particular case of a resource crisis, although probably the word "crisis" should be rejected as being unnecessarily emotion-charged. One of the responses to this perceived problem has been the growth of a new discipline called "resource analysis", spanning economics and the physical sciences, having as its purpose the study of the economic and technological structures determining the transformations and the rate of flow of resources from their natural state, through the production system, to final goods which are consumed. Taking energy as the particular natural resource to be considered in this paper, one of the basic questions concerns the rate at which the fixed stock of conventional energy is being exhausted; that this is important is based on the belief that the greater the length of time over which technological substitutions in the

energy field are allowed to occur, the less drastic will be the social and economic dislocations incurred.

The determinants of the rate of energy use by a society include the population size, a nebulous characteristic which might be termed "life-styles", population density, economic factors such as the relative price of energy commodities, and the per capita GNP, and technical factors such as energy production technology (which affects the energy needed to produce energy) and the set of energy-using technologies in the production system. In addition, in an economy which is highly open to international trade such as Canada's, where exports were roughly 20% of the Gross Domestic Product in 1966, the characteristics of external trade can also have significant effects on the rate of energy use. While it is straightforward to measure the direct exports and imports of energy commodities, a detailed energy analysis of all goods exported and imported can answer the following questions:

What is the impact on total energy use for a given year of:

1) the energy required to produce the energy which is exported directly,
2) the energy required to produce all other goods which are exported,
3) the energy which is "saved" as a result of importing goods (since these do not require domestic energy for their production),
4) the elimination of all trading in energy commodities?

These, and several others, are the questions addressed in subsequent chapters.

2. Analytical Tools

The vehicle of analysis for this study is an Input-Output based energy accounting model, "Input-Output" (abbr. I-O) being the name given to a class of economic models characterized by a highly disaggregated industrial sector and a fairly simple linear mathematical formulation. (The reader interested in a deeper understanding of the Canadian Input-Output models and the associated energy model should see references 1] and 2].) The Canadian I-O models identify over 200 industries producing over 600 commodities and are based on a set of commodity by industry accounts in current dollars of the inputs of goods to each industry and the production (outputs) of goods by each industry. The models are structural in the sense that technologies and market shares are explicitly represented, making the models sensitive to these technical and economic parameters. In this study time does not appear as an explicit variable, so that complex interactions between increasing demand, capacity constraints, and purchases of capital goods are not considered; however time appears implicitly in that many of the simulations involve phenomena transpiring over the course of a year, and also in that one year is the period over which the I-O accounts are measured, which tends to define which commodities are operational inputs for an industry and which are capital goods. An I-O model may be viewed operationally as a means of tracking the propogation of demand through the production system: the bill of goods demanded (referred to as the final demand) is produced by the appropriate industries, which themselves require

commodity inputs (making up the "intermediate" demand) which are produced by other industries, and so on; an I-O model is demand-driven, and the results produced are the <u>total</u> industry outputs required to satisfy demand, with the associated commodity production, as well as values of any parameters (for instance, manufacturing value added) which are related to the industry outputs.

The energy accounting model extends this Input-Output model by means of the assumption that the consumption of energy commodities in <u>physical units</u> is proportional to the value of industry production. The model is thus a hybrid of physical and value data, short of the "ideal" (from a resource-analytical point of view) of a physical I-O model, but eliminating some of the grosser distortions produced by inter - and intra-sectoral energy price variations. Specifying a final demand for a bill of goods in current dollars to the energy model produces as results the total intermediate use of energy commodities in natural units required to satisfy that demand. Thus, given a final demand consisting of the value of one automobile, the energy model calculates the energy used on the production line, plus the energy needed to produce the steel used in the auto, plus that for the rubber, the plastic, the glass, and so on, covering all the intermediate demand for energy. This energy accounting, for reasons which are data-constrained and methodological, does not attempt to assign any of the energy required to manufacture capital goods to the commodities which they produce.

The energy commodities, regarded as goods which can be delivered to final demand like any other, represent a special class of products in this modelling scheme: one dollar's final demand for gasoline, for instance, has an energy content calculable by means of a price per gallon and the enthalpy of combustion per gallon, but it also has an energy requirement which the model calculates as the total intermediate use of all fuels needed to satisfy that demand of one dollar. Energy analysis, being such a new field, requires at its basis a few new concepts and definitions of terms; the preceding sentence serves to define an essential distinction between energy content and energy requirements (this distinction has as its analogue that between final and intermediate demands in Input-Output parlance). The energy requirements to satisfy a given demand can be counted in different ways. In the first place, because the energy data base spans 15 energy types, the energy requirements could be expressed as a list of 15 quantities of fuels needed to satisfy demand, in gallons of gasoline, kilowatt-hours of electricity, and so on. By using an appropriate thermodynamic conversion, such as the enthalpy of combustion of fuels, this list could be expressed with all energy types measured in British Termal Units (abbr. BTU's). Adding these BTU's to produce a single "total energy requirement" figure requires careful interpretation: this number would be a gross requirement in the sense that it measures total flows of energy, both primary and secondary (secondary energy being that produced by a process of transformation from another form, e.g. coal as a primary fuel

may be converted to thermal electricity which is secondary); thus both thermal electricity and the coal and fuel oil which produced it are counted. If this double-counting of BTU's is avoided by adding only BTU's of primary energy, the resulting figure is referred to as the net energy requirement. Under either accounting procedure it should be noted that any "total BTU" figure for energy to satisfy final demand is implicitly a function of the energy use technology in all industries, since, for example, fuel oil and natural gas may have differing efficiencies in providing process heat in the Industrial Chemicals sector. However, as long as fixed technology is assumed it is possible to meaningfully compare energy requirements for different demands.

There are two final points about terminology. First, the term energy cost will be used interchangeably with energy requirement throughout this study, so that it is not a monetary figure which is being referred to when this term occurs. Secondly, energy intensity will refer to the energy cost of a given final demand normalized to one dollar of demand. Therefore the energy intensity of a particular commodity is the intermediate energy required to deliver one dollar of the good to final demand, and, similarly, for a bill of goods it is that energy needed to deliver these goods to final demand divided by the total value of the bill of goods.

The I-O based energy model is a functional combination of micro-structure and universality. This micro-structure is embodied in

the representation of technology in each industrial sector - there is a correspondence with theories of the firm in micro-economics in that these technologies are simple production functions. On the other hand the model is universal in the sense that it covers the universe of activities in the production system, from resource extraction to the provision of services. It is instructive to compare the model with energy accounting based on engineering "process analysis"; here it is possible to deal with technology at the level of the unit process and count the energy used through a chain of processes to the final good. While this allows the most detailed accounting of energy, certainly moreso than traditional I-O models, it lacks the comprehensiveness which typifies the latter. It is a feature of developed economies that virtually every industry is "connected" to every other, by which is meant that a final demand for the product of any industry necessitates some production by every other as the series of intermediate demands is met. There are feedbacks as well, so that in general a final demand for one dollar of a given commodity leads to more than one dollar's production, the excess serving as an intermediate input to other industries. These effects are clearly important when the question being asked is not "What amount of energy is used along a production chain?", but rather, "What is the total amount of energy used throughout the economy in delivering a commodity for final consumption?" And it is precisely these effects which the I-O approach captures. Moreover, the apparent trade-off of detail for comprehensiveness

in using the I-O based model is not an intrinsic limitation, for methodology exists (see reference 3) for breaking industries down into "activities" which could ultimately be unit processes.

Before proceeding with this paper there is a valid question which must be addressed, namely what relevance a study based on 1966 data has in 1976. The reason why 1966 was chosen is that, at the time of writing, this was the most recent year for which both complete Input-Output accounts and energy commodity accounts in physical units were available. The data indicate one clear change over time, this being in direct energy trade. In terms of a gross accounting of BTU's flowing across the border, Canada has historically been a net energy importer, a trend which continued until increasing exports of natural gas in the late 60's resulted in Canada being a net exporter of energy from about 1969 onward.

However, it is to be expected that one logical response to energy supply constraints will be a curtailment of energy exports, so that in the long term Canada would return to its historical trend of being a net importer of energy.

The implicit energy flows across the border, the energy costs associated with exported goods and the energy savings effected by importing goods, are determined by structural factors which have remained largely constant over the last decade. The structure of the bills of goods exported and imported is essentially in the proportion that each commodity forms of the total value; again, the historical trend has been that Canada has exported bills of goods which are weighted towards raw and semi-finished products, and imported bills of goods weighted towards secondary manufactured commodities. The Canada-U.S. auto pact was the major change in trade in the 1960's, the full effects of which were not felt until after 1966. Accordingly, the parameters relating to imports in the model have been ajusted to reflect post-auto pact conditions and other trends through the decade of the 1960's.

The important structural considerations in the production system are the fuel use technologies, and for several reasons these have remained relatively unchanged over the period in question. In the first place, the last major fuel substitution, that of petroleum products for coal, was largely complete by 1966. Secondly, and most importantly, the low pre-1973 prices of energy commodities led to energy being a small proportion of

total operating costs (in 1966 fuels and electricity in producers valuation accounted on average for less than 2% of industrial operating inputs), with the result that there was little incentive t o change fuel use technology. In addition, technological change is a gradual process, its reflection in the aggregate being the result of investment decisions by a large number of business establishments; in fact, at the time of writing it is not clear that the reaction to energy price increases dating from 1973 onward has led to many energy efficiency oriented responses more substantial than elimination of blatant wastes, in lighting for instance.

The final major structural parameter, which does not, strictly speaking, pertain to the industrial sector, is the proportion of electricity generated thermally. The thermal fraction was 19% in 1966 and increased to 21% in 1973; this is a minor variation which is largely compensated for by increasing efficiencies of transformation of fossil fuels to electricity; in any event, electric power is a "diffuse" good in the sense that there is not huge variation in the electricity intensity from commodity to commodity, so that relative (rather than absolute) co mparisons of energy costs should be largely independent of the thermal fraction.

For these reasons, which are all closely tied to the Input-Output hypothesis that structure is a persistent characteristic in the economy, one can argue that, with the exception of direct energy trade, an energy analysis of external trade for 1966 does indeed possess relevance a decade later.

3. <u>Summary of Results</u>

The body of this paper is dichotomous in its purpose, presenting on the one hand quantitative results relating to trade, and on the other, theoretical considerations pertaining to the treatment of external trade in energy analysis, together with mathematical results which clarify and explain the quantitative calculations. The following is a summary of the main points in the study.

(1) It is important to understand how external trade is modelled in the Canadian I-O system. Imports are related to final demand in such a manner that the level and pattern of imports is a function of Canadian technology and the propensity to import individual commodities. Exports are simply a final demand for goods like any other, with the exception that none of these goods can themselves have been imported.

(2) Depending on the choice of external trade parameters, there are four basic measures of energy intensities, each with their particular meaning and application.

(2) (i) The "closed economy" energy intensity assumes that all production, final and intermediate, occurs in Canada and that only domestic energy is used. As well as simulating a closed economy this measure may

serve as a proxy for the total energy required to deliver a given commodity to final demand.

(2) (ii) The "with import" energy intensity calculates total energy costs when intermediate and final imports are allowed in meeting demand. Because imported goods do not require any energy use in Canada in their production, this leads to a lower energy intensity than the previous definition. However, no distinction is made between domestic fuels and imported ones consumed in the production which does occur in Canada.

(2) (iii) The "domestic" energy intensity begins with the same calculation as the "with import" intensity, and assumes in addition that a fixed share of the domestic supply of each energy commodity is imported, allowing measurement of the total use of Canadian energy in meeting demand when imports are allowed. When no energy trade is permitted, which is a scenario explored in several portions of this paper, this measure is the same as the preceding one.

(2) (iv) All the above measures of energy intensity ignore the fact that imports and exports are intimately connected - in the long run trade must balance in value. To capture this effect the "balanced trade" energy intensity is implemented; as the name would

suggest, the assumption is that every dollar of trade is balanced. Therefore, to the "with import" energy intensity of the particular good must be added the energy cost of those exports required to balance trade. Given that, consistent with Canadian I-O methodology, the value and pattern of imports needed in the production of a given commodity is specific to that commodity, the key assumption in this calculation is, since exports are determined by external factors and by agents largely independent of those affecting imports, that the pattern of exports is the same for this "marginal" trade as was measured for 1966 in total. Although this mode of calculation is general enough to allow balancing particular patterns of imports by particular patterns of exports, it was felt that for the purposes of this study the foregoing assumption provided on average the best measure of total energy costs associated with production of any bill of goods.

(3) The first of the quantitative results is a measurement of the direct energy trade in BTU's for 1966 (a "quad" is one quadrillion - 10^{15} - BTU's):

Exports	Imports	Net Imports
1.149 quad	1.586 quad	0.438 quad

These figures measure crude oil at its refined product equivalent. Thus imports exceeded exports by 38%; when the Canadian primary energy needed to

produce this export energy, amounting to 0.048 quad, is counted as being <u>effectively</u> exported, imports exceed exports by 33%.

(4) In order to measure the effects of non-energy commodity trade a series of economy-wide simulations were performed, each with a particular assumption about external trade, but with the basis for comparison that the 1966 domestic demand for consumption and investment goods must be met in each case. In theoretical terms this is an assumption that the economy operates in such a manner as to meet domestic demand, and that external trade factors may be superimposed on this. The three simulations performed were to measure total domestic primary energy use for an economy that is open to external trade, with exports and imports at their 1966 level and pattern, one that is open except for trade in energy commodities, and one that is closed to external trade:

			Domestic primary energy (quads)	As % of (a)
(a)	Open economy	-	4.093	100%
(b)	Closed economy	-	4.416	108%
(c)	No direct energy trade	-	4.658	114%

The ranking of these three simulations gives considerable information about the structure of

external trade for Canada: Canadian non-energy commodity trade leads to Canada incurring greater energy costs for the goods exported than energy savings implied by the goods imported; given that there were net imports of direct energy in 1966, some of these net imports were effectively re-exported in closing the energy gap from (c) to (b), and only that portion from (b) to (a) substituted directly for domestic energy in meeting domestic demand.

(5) Since trade was within about 2% of being balanced in value in 1966, the energy gap between (c) and (b) above must occur because of differing energy intensities of the total bills of goods exported and imported. In fact, omitting fuel commodities and unallocated exports and imports, exports had a "closed economy" energy intensity of 98,782 BTU's per dollar while that of imports was 78,199 BTU's per dollar (exports were therefore, in total, 26% more energy intensive than imports). The energy use by industrial sector reveals the different commodity composition of these two bills of goods, with exports weighted towards primary industries and imports towards manufacturing industries. It is interesting to note that the closed economy energy cost of the total non-energy exports is 1.166 quads, which is extremely close to the energy content of the fuel and electricity exports, 1.149 quads.

(6) The balanced trade energy intensity, in addition to being an accurate estimate of total energy costs in the presence of external trade, allows a detailed commodity-by-commodity analysis of the effects of external trade on the production of individual commodities as well as bills of goods. In the same manner that comparing the "no energy trade" (c) and "closed economy" (b) simulations in section (4) provided a macro or global measure of the impact of non-energy trade on the Canadian resource base, comparing the balanced trade energy intensity (with no fuel trade) with the closed economy energy intensity for each individual commodity provides a micro-level measure of this same impact. This gives an in-depth view of the interaction between the structure of external trade and the technology of production of each commodity. Direct fuel trade is excluded in the balanced calculation because this is a category of trade which has changed greatly in the recent past, Canada making the transition from being a net importer to a net exporter of energy. Another motivation for this choice is the fact that the pure effect of non-energy commodity trade on the Canadian energy system is a distinct component of the energy analysis of external trade, therefore meriting individual study.

(7) Defining the "trade energy cost" of a commodity to be its balanced trade energy intensity minus its closed economy energy intensity, a positive trade energy cost indicates that more domestic energy is required to produce a given commodity or bill of goods in the presence of balanced trade than if there were no trade at all. This is the case with virtually all primary and service commodities in Canada, and, if imports directly to meet demand are assumed to constitute the same proportion of total demand as total imports do of total domestic supply, 56% of manufactured goods possess a positive trade energy cost; neglecting these direct imports this figure drops to 40%. By and large, commodities with a negative trade energy cost are closely tied to paper, steel, aluminum, chemicals, and ceramics - those with a positive trade energy cost include "natural" products (e.g. foods and natural textiles), high technology goods, transport and communications, motor vehicles, and primary non-ferrous metals (excluding aluminum). When weighted by the pattern of total domestic demand in 1966, by far the largest contribution to the net export of Canadian energy implicit in external trade in non-energy commodities was made by the demand for motor vehicles.

(8) Breaking total domestic demand down into nine broad classes, four of consumer expenditures, investment in

machinery and equipment, and construction, the latter two by both industry and government, and government current expenditures, each of these categories had a positive trade energy cost, with the exception of government construction. The largest positive trade energy costs were associated with investment in machinery and equipment and with consumer expenditures on durable and semi-durables. The high-growth categories of demand, consumer expenditures on services and government current expenditures, both exhibit relatively low positive trade energy costs, leading one to conclude that (all other things being equal) the direction in which demand has been moving over the last decade has not been towards increasing net implicit exports of energy.

(9) As the mathematical results described later in this section indicate, and as one would intuitively expect, the single most critical number in balanced trade calculations is related to the energy intensity of the export bill of goods, indicating that these goods merit closer inspection. The first point to note is that exports are dominated by a very few goods: out of over 500 commodities exported, 24 account for roughly 60% of the total energy requirements and value of this bill of goods - in fact, 11 of these 24, the raw and semi-finished goods, accounted for 53% of the total energy

intensity of exports. Therefore, in terms of energy costs, exports are dominanted by the raw and semi-finished commodities.

(10) This preponderance of raw and semi-finished goods in the energy cost of exports is due both to these gooods forming a large proportion of the total value of exports and to this class of commodities being relatively high in energy intensity. Investigating this latter fact in more detail leads to the initially suprising discovery that many raw materials are more energy intensive than the manufactured goods produced from them. The explanation for this seeming contravention of physical principles lies in the fact that the energy intensity as defined in this study is measured in BTU's per dollar. For any commodity this intensity could be defined as the ratio of two other intensities: the physical energy intensity in BTU's per natural unit, divided by the value added intensity, which is the price, in dollars per natural unit. The nature of the production chain is such that both energy added and value added are increasing functions of the degree of manufacture; however, the intermediate stages, including separation and refining, add much more energy than value (i.e. labour), whereas the secondary manufacturing stages add more value than energy. Under appropriate mathematical restrictions this phenomenon can lead to

manufactured goods being less energy intensive <u>per dollar</u> than raw and semi-finished goods.

(11) The relatively high energy intensity of Canadian exports is therefore related to these same goods not being particularly labour intensive. This means that, relative to a fixed total <u>value</u> of exports, exporting more manufactured commodities and less in the way of raw materials would lead to lower consumption of domestic energy for exports. This conclusion should be taken with the caveat that an increased capacity to produce manufactured goods for export possibly implies an increased capacity to produce these goods for domestic use, therefore decreasing Canadian reliance on imports of secondary goods. This could lead to a decrease in the level and role of international trade in Canadian economic activity.

(12) The final section of this paper presents in mathematical terms a variety of results which exhibit considerable explicative power. These may be summarized as follows:

(12) (i) Measuring the "closed economy" energy intensity of an import bill of goods as if it were any other bill of goods produced in Canada, the total trade energy cost associated with satisfying total domestic demand, given that trade is in balance, is directly

proportional to the closed economy energy intensity of exports less that of imports; thus if imports are less energy intensive than exports there is a positive trade energy cost.

(12)(ii) Defining the <u>import intensity</u> of any commodity or bill of goods to be the total value of imports needed to deliver one dollar to final demand, and the <u>energy cost per net export dollar</u> to be the "with import" energy intensity divided by one minus the import intensity, the balanced trade energy intensity may be written as the "with import" intensity plus the product of the energy cost per net export dollar of the bill of goods exported and the import intensity of the good or bill of goods in question.

(12)(iii) The trade energy cost, i.e. the balanced trade energy intensity minus the closed economy intensity, is given by the product of two factors, the first being the import intensity, the second the energy cost per net export dollar of the export bill of goods minus the closed economy energy intensity of the imports needed to produce the particular commodity or bill of goods. Thus if the imports specific to production of a given commodity are less energy intensive than total exports are per dollar of net exports, there is greater consumption of domestic energy when trade is

balanced then would occur in the absence of trade (cf. 12(i)).

12(iv) This energy intensity of imports associated with the production of a commodity is also equal to the energy cost per net export dollar of the bill of export goods for which the balanced trade and closed economy energy intensities are equal for this commodity (that is, for which there would be a zero trade energy cost). Plotting these figures for all commodities on a histogram gives some measure of the sensitivity of the balanced trade energy intensity calculation to variations in the energy cost per net export dollar of the bill of export goods. The histograms show that the calculation is more sensitive to decreases in this energy cost per net export dollar than to increases, i.e. the historical (1966) energy cost per net export dollar lies above the mode of the distributions.

12(v) The effects of direct imports (those going directly to meet final demand) on trade energy costs are as follows: treating the share that imports make up of a dollar of final demand as a variable for each commodity, the trade energy cost exhibits a positive rate of change with respect to this variable (i.e. it increases/decreases as the direct import share increases/decreases) if the energy cost per net

export dollar of the commodity in question is less than that of the total (historical) export bill of goods; for the opposite inequality there is a negative rate of change.

12(vi) Turning to a closer examination of the export bill of goods and its most important characteristic, the energy cost per net export dollar, we first observe that this figure, for a given bill of goods, is bounded above and below by the highest and lowest energy costs per net export dollar of the individual commodities constituting the bill of goods. Specifying the proportion that each commodity forms of one dollar of exports determines the energy cost per net export dollar; increasing the proportion of a particular commodity in this bill of goods, holding all other relative proportions constant, will increase the energy cost per net export dollar of the bill of goods if this commodity possesses a higher value for this characteristic than the original bill of goods. Of course, if the energy cost per net export dollar of the particular commodity is lower than that of the bill of goods originally, this produces an export bill of goods with a lower energy cost per net export dollar.

12(vii) For any bill of goods, the energy cost per net export dollar minus the closed economy energy intensity is

equal to the product of the import intensity divided by one minus the import intensity and the closed economy energy intensity of the bill of goods minus the closed economy energy intensity of the imports required for production of the bill of goods. Therefore, if an export bill of goods has a low import intensity its energy cost per net export dollar will tend to be close to its closed economy energy intensity.

12(viii) It is relatively straightforward to construct hypothetical export bills of goods for performing external trade "scenario" simulations. As the foregoing results would indicate, the key characteristic of exports from a balanced trade point of view is the energy cost per net export dollar. If one selects a particular subset of goods which are to be exported, and a target figure for the energy cost per net export dollar of the resulting bill of goods, then (as long as this target figures lies within the bounds imposed by the goods having the highest and lowest individual energy cost per net export dollar) the set of bills of goods having the target characteristic is the (non-empty, under the restriction in parentheses above) intersection of a hyperplane through the origin and the positive orthant in a Euclidean space. The interesting question from a policy point of view is therefore how

one selects a particular bill of goods lying in this set. One possible simulation, involving minimizing the deviation from a historical pattern of exports, is presented in this paper.

4. The Treatment of Imports in an Input-Output Based Energy Accounting Model

An energy accounting of a demanded bill of goods is essentially a measure of the total use of fuels and electricity as intermediate inputs to the processes of production involved in the satisfaction of that demand; moreover, this measure is necessarily limited to the Canadian economy. As a result, the use of imported commodities both as final and intermediate goods has large effects on the calculated energy cost of any bill of goods. Placing the energy system boundary at the Canadian border entails decreasing energy costs with increasing import values.

One possibility in calculating the energy cost of a given demand is to allow no imports, giving energy use in a hypothetical closed Canadian economy. This particular calculation is of interest for a couple of reasons: it measures the demand on Canadian energy resources which would follow from not engaging in external trade, and it serves as an estimate of the "total" energy cost (i.e. with the energy system boundary lying around the earth). An accurate accounting of the total energy cost of a good produced in Canada would measure energy use in Canada plus the energy cost of each imported commodity calculated in its country of origin, clearly leading to the necessity for a model of the global production system with explicit representation of trade flows. In the absence of such a model, the "no import" calculation is a good estimate of total use to

the extent that imported commodities are produced by technologies having energy use characteristics similar to their Canadian counterparts.

The effect of imports is handled in the Canadian Input-Output system and the associated energy model by assuming that the value of imports of each commodity is a fixed proportion of the domestic supply of that commodity. These import shares are based on historical data. From the point of view of model credibility, this methodology has the virtue that the value and proportions of imported goods are a function of the final demand because the domestic supply of each commodity is a function of final demand. In addition, these import shares may be varied for simulation purposes, as is in fact done later in this study.

One choice which this modelling scheme offers the user is whether or not to consider the import share of the final demand itself (hereafter referred to as "direct imports"). For much comparative work, such as comparisons of the energy intensities of individual commodities, it is useful to ignore direct imports, thus making the calculation on a common basis of $1 of domestic demand which must be met by domestic production.

The effect of modelling imports in this manner on the measurement of energy costs is a significant reduction of the same relative to the "no import" calculation. This occurs because imported goods now have zero energy requirements. However, the energy model in this form still measures the energy content of imported fuels even through they come free of energy

requirements, which suggests a third possible measure of energy requirements: by subtracting away the import share of each fuel used it is possible to define the <u>Canadian</u> energy requirement of a given final demand, the net use of Canadian resources.

There is a slight conceptual problem with this last definition of energy requirements, in that refined petroleum fuels produced from imported crude oil should not be counted as a use of Canadian resources. The problem is that, because its physical properties preclude its direct use as a fuel, crude petroleum is not counted as an element of either gross or net energy requirements. The most reasonable way around this difficulty is to subtract away the BTU's of imported crude oil, multiplied by the average efficiency of conversion of crude to refined petroleum fuels, to arrive at the Canadian energy requirement.

Table 1 presents a comparison of these three measures of energy requirements for an average 1966 automobile, and Table 2 gives a comparison of energy intensities based on the "no import" and "no direct import" calculations for a variety of commodities.

Table 2 clearly shows the large variation possible in both the absolute difference between the two energy intensities and in the percentage change (from 1% to 59% relative to the "no import" calculation), brought about by the varying levels and make-up of the bills of imported goods necessary to the production of these commodities.

TABLE 1

ENERGY REQUIREMENTS OF DEMAND FOR ONE AVERAGE 1966 AUTOMOBILE

		Millions of BTU's (net)
1. Energy Requirements - no imports	-	144
2. Energy Requirements - no direct imports	-	58
3. Canadian Energy Requirements - no direct imports	-	29

TABLE 2

ENERGY INTENSITIES OF SELECTED COMMODITIES

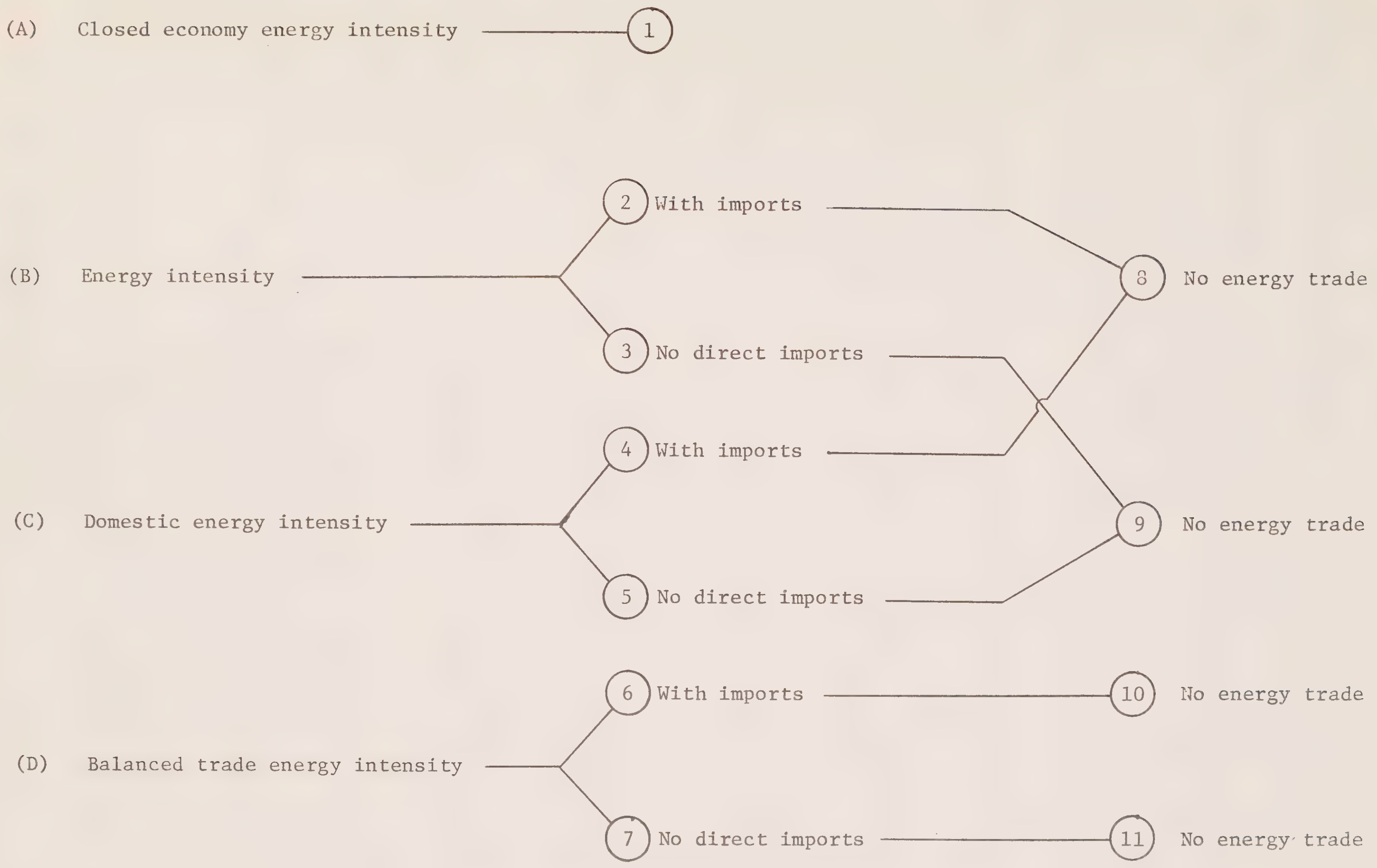
(Thousands of BTU's Per Dollar Demand)

Commodity		(1) No Import Energy Intensity	(2) No Direct Import Energy Intensity	(3) Difference As % of (1)
00200	Sheep and Lambs	57.200	51.511	9.95%
03900	Natural Gas	108.918	107.214	1.56%
05800	Margerine & Shortening	56.551	45.712	19.17%
08100	Vegetables, Canned	62.919	49.088	21.98%
10600	Oilseed, Meal and Cake	72.770	29.830	59.01%
12000	Alcoholic Beverages, Distilled	51.765	45.949	11.23%
13000	Tires & Tubes, Automobile	81.265	55.283	31.97%
15800	Papermaker's Felt	54.990	33.724	38.67%
16000	Man Made Fibres	134.007	113.756	15.11%
23200	Fine Paper	161.389	153.438	4.93%
26200	Steel Castings	206.563	189.862	8.09%
29400	Nickel & Alloy Fabricated Mat.	86.826	70.771	18.49%
31300	Containers & Bottle Caps, Metal	92.890	75.226	19.02%
35900	Industrial Furnaces, Kilns, Ovens	56.218	39.448	29.83%
38900	Snowmobiles, Etc.	52.743	34.020	35.50%
40200	Electronic Equipment Components	32.026	21.236	33.69%
41500	Cement	358.219	354.182	1.13%
45000	Fertilizers	136.451	117.960	13.55%
49000	Vinylchloride Monomer	242.960	224.927	7.42%
55500	Measure & Control Instr.	45.234	29.433	34.93%
59100	Urban Transit	55.269	53.318	3.53%
60600	Retailing Margins	44.376	41.469	6.55%
60810	Imputed Service, Banks	13.389	12.048	10.01%
63500	Laboratory Equipment & Supplies	59.279	26.852	54.70%

Finally, there is a fourth possible definition of energy intensities when imports are allowed, based on the concept of balancing import and export dollars, which will be presented later in this paper. Referring to this last as the "balanced trade" energy intensity, and re-naming the "no import" calculation to be the "closed economy" energy intensity, Diagram A depicts the 11 energy intensities defined in this paper. The final four calculations assume no trading in energy commodities, so the pure effect of trade in all other commodities may be measured; it should be clear that the energy intensity (B) is equal to the domestic energy intensity (C) under these conditions.

DIAGRAM A

Possible Calculations of Energy Intensities



5. The 1966 Energy Trade Balance

(i) Direct Energy Trade

The direct term in the energy analysis of external trade for Canada is simply a measure of the flows of energy commodities across the border, measured in physical units. Table 3 summarizes this trade.

In 1966, therefore, Canada was a net importer of energy to the extent of 468,355 billion BTU's. However, to correctly measure the real impact of direct energy trade, a couple of adjustments must be made to these figures. First, to be consistent with energy accounting concepts used in the rest of this paper, crude petroleum should be measured in terms of its average refined product equivalent. Secondly, the production of the above fuels for export is a process requiring energy, and this energy is in effect exported as well. Strictly speaking, this "energy to produce energy" is not a direct element in energy trade, but, because this information will be buried in any more complete treatment of trade in all commodities, it is useful to present it explicitly at this point.

By specifying the fuel exports from Table 3 in dollar values, this vector of demand can be energy costed using the energy model. This calculation was performed under the following assumptions:

i) the accounting concept used is that of the Canadian net energy requirement described in the previous section;

ii) only hydro electricity is exported;

TABLE 3

DIRECT ENERGY TRADE

(Billions of BTU's)

		Exports	Imports
1.	Crude petroleum	735817	920031
2.	Coal	31707	424156
3.	Natural gas	436136	45053
4.	Electricity	15004	10980
5.	Coke	2138	14507
6.	Gasoline	2800	13402
7.	Fuel Oil	6572	311945
8.	Liquefied Petroleum Gases	41560	15
	TOTAL	1271734	1740089

TABLE 3(a)

DIRECT ENERGY TRADE

(Billions of BTU's)

		Exports	Imports
1.	Total energy (from Table 3)	1271734	1740089
2.	Transformation losses for crude petroleum	-122881	-153645
3.	Canadian net energy requirement of exported fuels	48145	-
	ADJUSTED TOTALS	1196998	1586444

Net Imports	389446
Net Imports (excluding 3)	437591

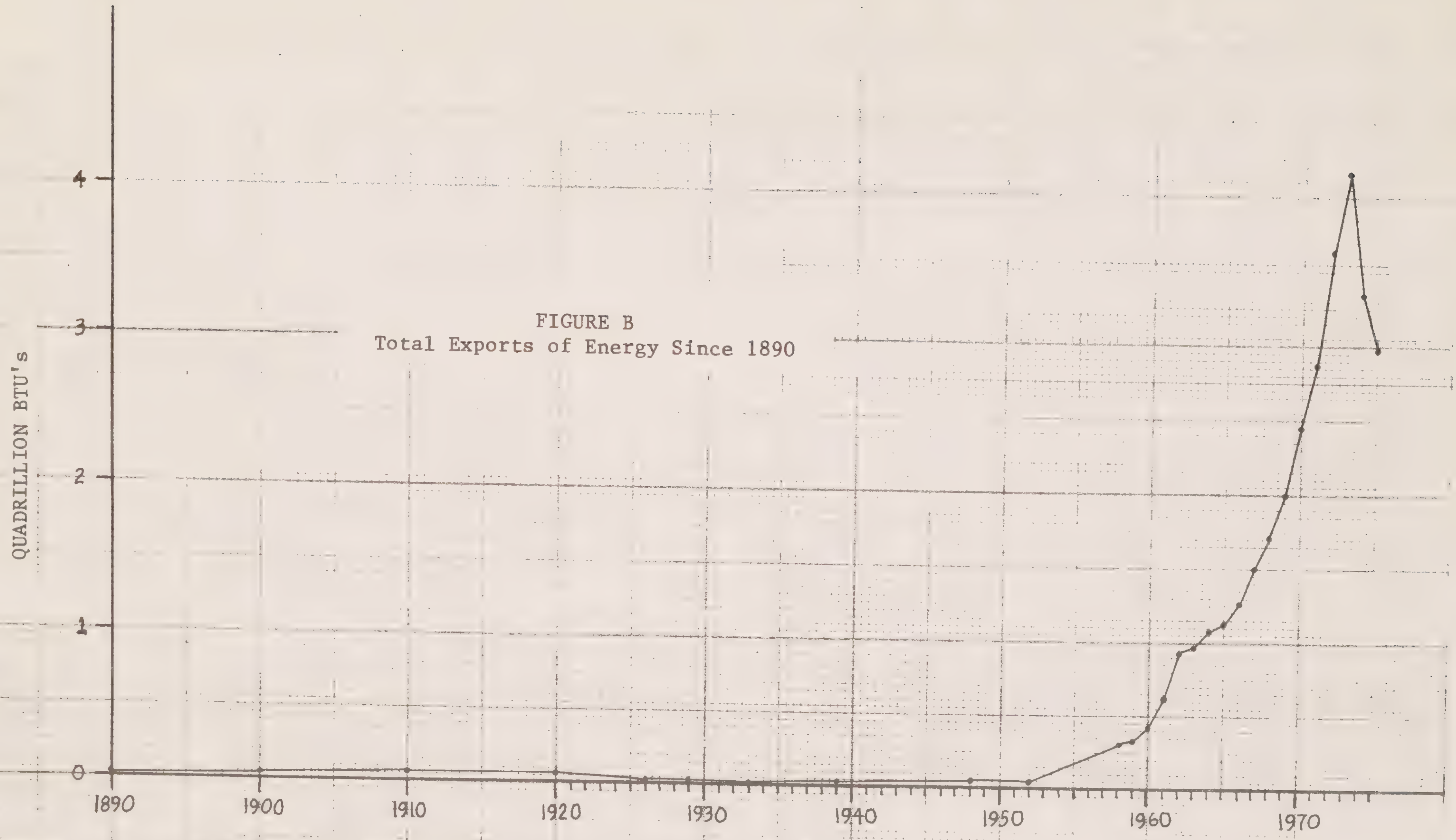
iii) energy producing industries do not employ imported fuels in the production of export energy.

Assumption (ii) is rather conservative - it attempts to capture the fact that in the largest electricity producing provinces thermal power is used only for peaking, and is therefore not likely to be exported.

The results of these adjustments to the direct energy trade balance are given in Table 3(a).

To put the 1966 net imports of direct energy in a historical context, Figure A shows the Canadian net imports of energy since 1890, the quantity measured being the BTU equivalents of all energy goods (primary and secondary) crossing the border, crude oil included. As noted in the section on analytical tools, Canada made the transition from being a net importer to being a net exporter of energy in 1969, with the balance edging towards equilibrium in 1975. Figure B gives the total BTU's exported since 1890. What is of interest here is that, assuming the linear interpolation between sparse data points is correct on average, the cumulative exports for the five years 1971-1975 are only slightly less than the cumulative exports for the years 1890-1970.

FIGURE B
Total Exports of Energy Since 1890



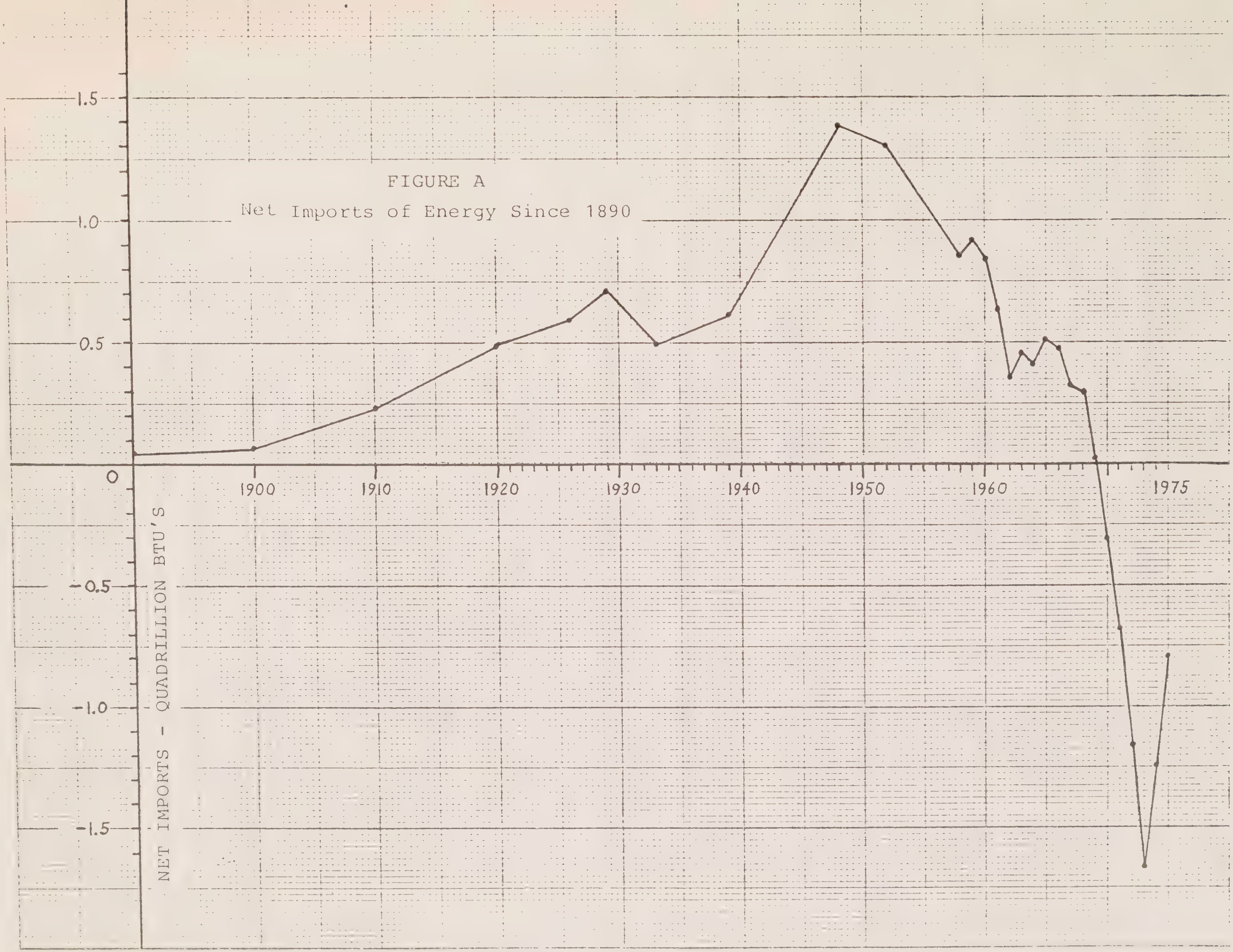


FIGURE A
Net Imports of Energy Since 1890
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
1900
1910
1920
1930
1940
1950
1960
1975
NET IMPORTS - QUADRILLION BTU'S

(ii) Direct and Indirect Energy Simulations

The problem of measuring the effects on the Canadian energy system of external trade in all commodities is considerably more complex than the measurement of direct flows of fuels and electricity, although the general direction of the analysis is clear: importing goods in the satisfaction of a certain domestic demand will decrease the use of Canadian energy, and exporting goods will increase it, this latter occurring since exports represent an increase of demand beyond the level of simply satisfying domestic needs.

The energy model, through appropriate selection of demands and parameters, allows the simulation of a Canadian economy which is closed or open with respect to external trade. The methodology used to measure the overall energy impact in these simulations is as follows:

1) Each simulation involves the same basic final demand, consisting of the 1966 values of consumer expenditure, investment in construction of new plant, investment in machinery and equipment, and government current expenditures. This is essentially the historical demand of goods for domestic consumption.
2) The "open economy" simulation adds to the above demand the 1966 bill of goods exported, and calculates the direct and indirect imports according to the historical import shares.
3) The "closed economy" simulation involves only the above demand, and all import shares are set to zero.

4) The energy impact in each simulation is calculated as the total net BTU's of Canadian energy for both final and intermediate use.

The results of these two simulations are presented in table 4. These figures require some explanation. First, the total energy use in the open economy will not agree exactly with any published figures for total energy used because this is a measure of net Canadian energy flows corresponding to a final demand which is not exactly the "balancing" final demand, the particularly volatile category of inventories having been omitted. Secondly, the difference in final use of energy for these two simulations is not equal to the total exports of energy as shown in the preceding section both because this is a net accounting and because there are direct imports of fuels for domestic final consumption in the "open" simulation.

In summary, the simulations show that engaging in international trade in 1966 resulted in a "saving" of 322,913 billion BTU's of Canadian primary energy (7.9% of the total) relative to a hypothetical closed Canadian economy. It is interesting to note that closing the economy has the effect of redistributing the energy flows as well as increasing them; flows of Canadian primary energy to final demand are increased by the elimination of direct fuel imports, but decrease overall as a result of the elimination of exports, whereas intermediate flows are increased significantly by elimination of imports of all commodities, both energy and non-energy.

TABLE 4

OPEN AND CLOSED ECONOMIES - CANADIAN NET ENERGY USE

(Billions of BTU's)

Simulation	Final Use	Intermediate Use	Total
1. "Open economy"	2819267	1273408	4092675
2. "Closed economy"	1821407	2594181	4415588
Difference (2) - (1)	-997860	1320773	322913

TABLE 5

ENERGY SELF-SUFFICIENCY - CANADIAN NET ENERGY USE

(Billions of BTU's)

Simulation	Final Use	Intermediate Use	Total
1. "Open economy" (No energy trade)	1821407	2836656	4658063
2. "Closed economy"	1821407	2594181	4415588
Difference (2) - (1)	0	-242475	-242475

This latter distinction between energy and non-energy commodities raises the possibility of a third simulation, since the direct trade in fuel commodities, which is measured in the preceding section, masks the effects of trade in all other commodities. To capture this information a third open economy simulation was run, under the same assumptions as the previous "open" run, but now with the exports of fuels set to zero and the import shares of fuels also set to zero. This could be viewed as a simulation of an energy self-sufficiency policy, with all other trade relationships remaining the same. The results are shown in Table 5, together with a comparison with the closed economy results.

Thus Canadian external trade in non-energy commodities only, according to the 1966 patterns of imports and exports, results in an energy "loss" of 242,475 billion BTU's relative to the option of not trading at all.

This result could also serve as an approximation to an energy balanced trade simulation in which the policy objective is to balance energy trade in BTU's rather than in value. Under whatever interpretation, Table 5 indicates that the energy cost of the non-energy goods which Canada exports exceeds the energy cost foregone by employing the non-energy goods which Canada imports.

Tables 4 and 5 together imply the following ranking in terms of total Canadian net energy required to satisfy demand for goods and services:

		Billions of BTU's
(1)	Energy Self-Sufficiency	4658063
(2)	Closed Economy	4415588
(3)	Open Economy	4092675

The fact that (1) and (3) differ by more than the net imports calculated in the preceding section is due largely to the energy requirements of this formerly imported energy which must now be produced in Canada, fuels being relatively highly energy intensive goods. These results show that Canada's trade in non-energy commodities makes her, in terms of these commodities, effectively a net exporter of Canadian energy resources, and that it is only the net <u>direct</u> imports of energy which more than cancel this effect.

These three simulations can in general present information on the role and importance of energy and non-energy trade in their impact on the Canadian resource base, depending on the particular ranking obtained. To simplify the following exposition, assume, because Canada exports non-energy goods which are generally primary, raw or semi-finished, and imports non-energy goods which are secondary manufactured, that the Canadian energy cost of exported commodities exceeds the Canadian energy saving implied by imported commodities. (This assumption is examined in more detail in the next section.)

Using acronyms associated with simulations (1), (2) and (3) of the preceding paragraph, we therefore have assumed that

$$ESS > CE ,$$

and shall refer to this as an assumption of net <u>implicit</u> exports of Canadian energy.

There are five possible rankings in total Canadian primary energy use:

1. $OE > ESS > CE$
2. $OE = ESS > CE$
3. $ESS > OE > CE$
4. $ESS > OE = CE$
5. $ESS > CE > OE$

Each of these rankings can now be interpreted as follows:

1. Canadian energy subsidizes both net implicit exports and further net direct exports of energy.
2. Direct energy trade is balanced in BTU's, so that the net effect is that Canadian energy, as well as meeting domestic needs, supplies energy for the net implicit exports as well.
3. In this case there are net direct imports of energy which are, in effect, wholly re-exported in covering a portion of the net implicit exports, the balance being Canadian energy.
4. Total energy balanced trade is achieved, in the sense that net direct energy imports equal net implicit energy exports. Again, the net direct energy imports may be viewed as being wholly re-exported.
5. A portion of the net direct energy imports is re-exported as net implicit energy exports, the balance actually substituting for Canadian energy in meeting domestic demand.

The last inequality is, of course, the one observed for 1966, and points out the fact that net imports of energy do not substitute directly "one-to-one" for domestic energy in meeting domestic demand in an open economy, since a significant proportion of this energy is effectively re-exported in non-energy trading.

This chapter is best summed up by borrowing some numbers from the sections preceding and following to give table 5(a). Here the "open economy" simulation is broken down into its component parts, these being the flows of energy supporting external trade and domestic economic activity. The energy saved as a result of importing is represented explicitly as a negative use of Canadian primary energy.

TABLE 5 (a)

COMPOSITION OF TOTAL CANADIAN PRIMARY ENERGY CONSUMPTION IN AN OPEN ECONOMY

		(quads)
1.	Direct final demand for energy	1.82
2.	Energy cost of total domestic demand	2.59
3.	Direct energy exports	1.15
4.	Energy cost of direct exports	0.05
5.	Energy cost of non-energy exports	1.17
6.	Direct energy imports	- 1.59
7.	Energy cost of direct imports	- 0.12
8.	Energy cost of non-energy imports	- 0.98
	Total	4.09

6. Export and Import Energy Intensities

Note: This section and the previous one employ a variant of the energy model which gives a very detailed calculation of transformations to thermal electricity. The methodology gives rise to a somewhat (i.e. about 5%) higher energy intensity for most goods than the computationally less complex calculations employed in the rest of the study. For instance, exports which in this section are calculated to have a closed economy energy intensity of about 99 thousand BTU's per dollar will later be assigned an intensity of about 93 thousand BTU's per dollar.

In this section we examine in more detail the reasons for the difference between the "energy self-sufficiency" and "closed economy" simulations. Using the energy model it is possible to define the Canadian net energy intensity (recalling that this is the total use of primary Canadian energy to deliver $1 of a good to final demand) of each commodity in a closed economy; these energy intensities may then be used to calculate, in a straightforward manner, the total energy cost of any bill of goods. There are two properties of this set of energy intensities which impinge upon what follows. The first point is that, generally speaking, each commodity has a unique energy intensity, and secondly, there is an almost 100-fold variation in these intensities across the set of non-energy commodities.

The bill of goods demanded in the "closed economy" simulation was the 1966 domestic demand for goods and services, e, which, when each entry was multiplied by its closed economy energy intensity and the results summed, produced the total primary Canadian energy requirement derived in the previous section. It may be shown (and the mathematical details will be given later) that the "energy self-sufficiency" result can be obtained by performing the same calculation where now the bill of goods demanded is e + x - m, where x and m are respectively the bills of goods exported and imported in 1966, with all energy commodities set to zero. Note that this is an explicit statement of the idea that the energy savings implicit in non-energy imports should be measured as that energy which would have been required to produce the imported goods in a hypothetical closed economy. Imports therefore lead to foregone energy costs.

The practical importance of the fore-mentioned properties of the set of energy intensities is, given that the total value of exports and imports was roughly in balance (and in fact the 1966 Input-Output accounts show total imports exceeding total exports by 2.2%), that it is the relative proportions of the particular commodities that are imported and exported that determine the magnitude and the sign of the change in primary Canadian energy between these simulations. Therefore, to capture this effect, a closed economy energy model calculation was performed on $1 worth of exports, with commodities distributed according to their 1966 proportions, and similarly for $1 worth of imports.

The results of these calculations are shown in Table 6 but, as usual, some caveats and explanations are necessary before proceeding with the interpretation. First of all, non-competitive imports were not included in the import run, and other exclusions common to both were the energy commodities and a "commodity" which is really an accounting convenience called "unallocated exports and imports". And secondly, the figure for "gross industrial disposition" is the sum of the BTU's of all fuels and electricity which were consumed by the production system, exclusive of transformations from one energy type to another.

Table 6 makes explicit what Table 5 would lead one to expect - dollar for dollar, the Canadian net energy requirements of the goods exported are 26% higher then the requirements of those imported. In terms of individual energy types, the exports use higher relative proportions of electricity and fuel oil, and the imports higher relative proportions of coal and coke (note that neither of these pairings are independent because these are gross fuel measures). However, it is the variation in sectoral relative proportions which reflects more clearly the differing commodity mix of imports and exports. Because non-energy use of fuels is confined to the manufacturing sector, the total proportion of energy use in manufactured imported goods is over 10% greater than for exports. On the other hand, the primary industry share is roughly 7% higher in exports than in imports, along with a 3% higher transportation sector use, this latter effect being due to the fact that transportation margins can be

TABLE 6

EXPORT AND IMPORT ENERGY INTENSITIES IN A CLOSED ECONOMY

(BTU's per dollar)

Direct Energy Trade Excluded

Disposition:	EXPORTS		IMPORTS	
(1) Gross industrial disposition	90884		71084	
(2) Use by sector:	Disposition	As % of (1)	Disposition	As % of (1)
Non-energy use	7292	8.0%	12600	17.7%
Energy supply industries	9525	10.5%	7118	10.0%
Primary industries	13179	14.5%	5513	7.8%
Manufacturing industries	48032	52.9%	38034	53.5%
Commercial, transport & other	12817	14.1%	7796	11.0%

Energy Requirements:	EXPORTS		IMPORTS	
(3) Canadian net energy requirement	98782		78199	
(4) Gross fuel requirement	106918		89222	
(5) Use by fuel:	Use	As % of (4)	Use	As % of(4)
Coal	23072	21.6%	24212	27.1%
Natural gas	17495	16.4%	16443	18.4%
Electricity	21404	20.0%	13570	15.2%
Coke	4966	4.6%	8915	10.0%
Gasoline	6315	5.9%	3722	4.2%
Fuel Oil	30617	28.6%	19649	22.0%
L.P.G.	1148	1.1%	1235	1.4%
Still gas	1400	1.3%	1139	1.3%
Purchased Steam	498	0.5%	338	0.4%

measured for domestically produced goods but not for imported ones.

The basic result of this analysis is that, owing to the particular commodity mix of Canada's exports and imports, on average the Canadian net energy cost of every dollar of exports is twenty thousand BTU's larger than the saving implied by a dollar of imports. Given the very large role that external trade plays in Canada's total economic activity, this is not inconsiderable. The broad pattern of the bills of goods exported and imported is fairly stable over time, by which is meant that Canada has been and will likely continue to be an exporter of raw and semi-finished goods and an importer of manufactured products.

7. The Balanced Trade Energy Intensity

(i) Motivation and Methodology

As indicated in an ealier section of this paper, the openness of an economy to external trade considerably complicates the assignment of energy costs to commodities. However, there are several characteristics of external trade which are reflected in an input-output model, and which serve to provide another means of defining energy intensities. The first point to note is that, in the long run, trade must be in balance in value. The second is that exports represent a demand whose make-up is largely exogenously determined, responding to foreign markets and not to domestic ones. And finally, given that there is an exogenously explained propensity to import a certain share of the domestic supply of each commodity, the level and pattern of commodity imports is a function of the bill of goods domestically demanded. In an I-O model, this functio n is represented by the input and output relationships, which in turn are a representation of the set of technologies extant in the economy.

The problem with considering imported goods to be completely free of energy requirements, which leads to one possible definition of energy intensities, is that this is not a complete analysis. The implication of a long term trade balance is that, on average, every dollar of imports must be balanced by a dollar of exports, and these exported goods have associated energy requirements. It is possible, therefore, to conceive of a balanced trade energy intensity of a given commodity which is

the sum of the energy intensity "with imports" and the energy cost of export commodities, distributed according to their externally defined proportions, equalling in total the value of imports necessitated by production of the given commodity. The calculation of such an intensity is complicated somewhat by the fact that producing goods for export is itself a process requiring imported goods.

Given that the assumptions of fixed import shares and balanced trade value are acceptable, this calculation of intensities is most sensitive to the pattern of commodities exported. Noting that the bill of goods exported is determined by external factors, and in addition, that there is no *a priori* reason for the total imports necessary for the production of a specific commodity to be balanced by a particular collection of exports, the best assumption, on average, in calculating this intensity is that the pattern of a marginal quantity of exports is the same as that of the measured total exports for a given time period, in this case 1966. The energy intensity using this pattern of exports and defined according to the methodology described above will henceforth be referred to as the balanced trade energy intensity.

(ii) Energy Self-Sufficiency Calculations

It is clear that direct energy trade will have large effects on the balanced trade energy intensities, for on the one hand imported fuels will substitute for Canadian energy resources, and on the other the bill of goods exported will include Canadian energy. In terms of the parameters of the model, the sensitive areas will be the import shares of energy commodities and the proportions that energy commodities constitute in an average dollars's worth of exports. Out of the myriad possible simulations the case of energy self-sufficiency was deemed of sufficient interest to warrant further investigation; operationally, this option amounts to setting all the above-mentioned parameters in the model to zero.

Two concepts which will prove useful in the discussion which follows are: (i) the import intensity of a commodity; and (ii) the energy requirement per net export dollar. The first of these is defined and calculated in a manner exactly analogous to the energy intensity. The import intensity of a good is the total value of imported commodities used as intermediate inputs necessary to deliver $1 of the good to final demand. The second concept relates to a bill of goods which is to be exported. As noted earlier, production of goods for export requires the use of imported commodities, and so it is possible to define the value of <u>net</u> exports as the value of the bill of goods exported less the value of the associated imports. Dividing the energy requirements of the exported commodities by this figure gives

TABLE 7

1966 EXPORTS

(Excluding Energy Trade)

Energy cost of $1 worth of exports, with imports of intermediate goods allowed, in BTU's per dollar.

Value of exports	–	$1.000
Value of imports	–	$0.134
Net exports		$0.866

	Per $ of Exports	Per $ of Net Exports
Net Energy Requirement	80064	92453

the energy requirement per net export dollar. Table 7 presents this calculation for the 1966 pattern of exports.

The concepts defined in the previous paragraph are important because the mathematics of the energy model (presented briefly at the end of this paper) show that the balanced trade energy intensity of a commodity is given by the formula

$$\varepsilon^{m} + \beta z \qquad (1)$$

where ε^{m} is the net energy intensity of the commodity with imports allowed, β the net energy requirement per net export dollar (92,453 BTU's in 1966, as calculated in Table 7), and z the import intensity of the commodity. We shall return to this formula after discussing some of the results calculated on this basis.

The "direct and indirect" energy simulations presented earlier in this paper gave a total measure of the impact on energy resources of an open economy which is energy self-sufficient as compared to a closed economy. As well as providing what is arguably the best estimate of the total energy required to satisfy a particular demand, the balanced trade energy intensity allows a comparison of these two scenarios in great detail, by means of a commodity-by-commodity comparison of intensities. It is therefore possible to see which commodities incur a higher use of energy (per dollar of demand) as a result of engaging in external trade, and which lower. In addition to the interest this information may have <u>per se</u>, it facilitates a qualitative analysis of the energy impact of various categories of final

demand, and provides some insight into the sensitivity of the energy trade balance to variations in, for instance, the energy cost per net export dollar of the goods exported. Ultimately, comparing "closed economy" and "balanced trade" energy intensities of individual commodities is of theoretical interest because it represents the decomposition of the closed economy and energy self-sufficiency macro simulations of chapter 5 into their most elementary components.

There are two basic variants possible in calculating the balanced trade energy intensity, which lead to large differences in magnitude but not necessarily in rank with respect to the closed economy energy intensity. One possibility is to allow direct imports, so that one dollar of total demand for a commodity is met partly by a direct import, the rest being met by domestic production. On average the best assumption is that direct imports form the same proportion of a dollar of demand as they do of total domestic supply; thus the direct import share is the same as the total import share. Although it is highly sensitive to this assumption, the balanced trade energy intensity with imports (direct and intermediate) is "historically" correct, in that, when weighted by the measured 1966 total demand, it gives the total energy use for an energy self-sufficient economy as measured in chapter 5. The second possibility is to allow no direct imports in calculating the balanced trade energy intensity, meaning that each dollar of demand must be met by Canadian production. Because this measure of energy intensity for a given commodity is effectively

independent of the import share, and any assumptions about the direct import share, of the good in question, it may be viewed as a more intrinsic calculation of the balanced trade energy intensity, reflecting greater sensitivity to the technology of production rather than the individual import share.

The large quantity of data available dictates that only a representative sample should be presented in comparing closed economy and balanced trade intensities, and this is accordingly presented in Table 8, for the same commodities as appear in Table 2. For purposes of brevity the following acronyms are introduced at this time:

CEEI - closed economy energy intensity

BTEI - balanced trade energy intensity

TEC - trade energy cost

TEC = BTEI - CEEI

As one would expect, Table 8 indicates that commodities which have no import share (such as natural gas and service goods in this table) have the same BTEI by either mode of calculation. To indicate the rankings of CEEI and BTEI for broad categories of commodities, we turn to Table 9.

It is clear, therefore, that BTEI exceeds CEEI for both measures in virtually the entire categories of primary and service commodities. The chief differences occur in the secondary classification, with the relative rankings for the two measures being almost exactly opposite. This is sufficient to ensure

TABLE 8

CLOSED ECONOMY AND BALANCED TRADE ENERGY INTENSITIES FOR SELECTED COMMODITIES

(Thousands of BTU's Per Dollar Demand)

Commodity		CEEI	BTEI "with imports"	TEC "with imports"	BTEI "no direct imports"	TEC "no direct imports"
00200	Sheep & lambs	57.20	59.50	2.30	57.41	0.21
03900	Natural gas	108.92	109.72	0.80	109.72	0.80
05800	Margerine & shortening	56.55	58.75	2.20	58.37	1.82
08100	Vegetables, canned	62.92	69.14	6.22	65.50	2.58
10600	Oilseed, meal and coke	72.77	95.01	22.24	97.00	24.23
12000	Alcoholic Bev., distilled	51.77	66.55	14.79	52.90	1.14
13000	Tires & tubes, automobile	81.27	72.89	-8.37	72.13	-9.13
15800	Papermaker's felts	54.99	62.42	7.43	61.74	6.75
16000	Man-made fibres	134.01	116.71	-17.30	129.24	-4.77
23200	Fine paper	161.39	152.60	-8.79	160.41	-0.98
26200	Steel castings	206.56	193.79	-12.78	205.29	-1.27
29400	Nickel & alloy fab. mat.	86.83	89.51	2.68	85.94	-0.89
31300	Containers & caps, metal	92.89	89.13	-3.76	88.83	-4.06
35900	Ind. Furnaces, kilns, ovens	56.22	77.17	20.95	56.29	0.07
38900	Snowmobiles, etc.	52.74	53.27	0.52	52.35	-0.39
40200	Electronic equip. components	32.03	71.47	39.44	36.90	4.87
41500	Cement	358.22	352.22	-6.00	358.39	0.17
45000	Fertilizers	136.45	123.87	-12.58	131.21	-5.24
49000	Vinylchloride monomer	242.96	223.63	-19.33	236.35	-6.61
55500	Measure & control instr.	45.23	72.55	27.32	46.10	0.87
59100	Urban transit	55.27	55.34	0.08	55.34	0.08
60600	Retailing margins	44.38	44.86	0.48	44.86	0.48
60810	Imputed service, banks	13.39	13.88	0.49	13.88	0.49
63500	Laboratory equip. & supplies	59.28	81.19	21.91	81.19	21.91

TABLE 9

Closed Economy and Balanced Trade Energy Intensities, by Category

Classification	With Imports: CEEI > BTEI	With Imports: BTEI > CEEI	No Direct Imports: CEEI > BTEI	No Direct Imports: BTEI > CEEI
Primary	10	41	5	46
Secondary	238	298	324	212
Service	4	44	5	43
TOTALS	252	383	334	301

Number of Commodities (column group heading)

that for the "with imports" calculation, the number of commodities for which BTEI exceeds CEEI is considerably larger than for the opposit e ranking. In the "no direct import" calculation the two rankings are roughly equal in total number of commodities; in fact, eliminating goods which are indistinguishable (reference 2 contains theorems implying that if a set of commodities is produced by an industry which is their sole producer, then these commodities have exactly the same energy intensity, making them indistinguishable - an example would be many of the industrial chemicals) the two totals agree almost exactly. Recalling that (the trade energy cost) TEC = BTEI - CEEI, Table 9 gives information about the sign of TEC, but not its magnitude. This will be the subject of the next section.

The following list of goods represents those general classifications of commodities for which CEEI exceeds BTEI (by either measure); that is, commodities for which engaging in external trade produces a real energy saving in the sense that demanding for domestic consumption a dollar of these goods costs less energy in the presence of trade than if the economy were closed to trade. This implies that, fixing import shares and the pattern of exports, if domestic final demand shifted towards these goods and away from those for which BTFI exceeds CEEI then the net exports of energy implicit in our non-energy commodity trade would decrease.

These are all fairly closely tied to steel, aluminum, and chemicals:

1. Rubber products
2. Man-made fibres and fabrics
3. Paper products (excluding printed matter)
4. Steel products, primary and fabricated
5. Aluminum products
6. Fabricated non-ferrous metals
7. Miscellaneous steel and metal products (boilers, culvert pipe, etc.)
8. Glass and ceramic products
9. Liquid hydrocarbon fuels and by-products
10. All chemicals and chemical products
11. Plastics

In addition, CEEI exceeds the "no direct import" BTEI for the following goods:

12. Tools, hand and power
13. Miscellaneous mechanical goods (heating equipment, conveyors, etc.)
14. Motor vehicle parts and hardware
15. Boats and ships
16. Miscellaneous manufactured goods (sporting goods, phono records, etc.)

Aside from the preponderance of primary and service commodities that are energy losers (i.e. for which BTEI exceeds CEEI) in terms of balanced trade, as noted in Table 9, some comment should be made about the other commodities for which this is true. The first broad class might be called "natural" goods, consisting of processed foods, beverages, tobacco products, leather goods, natural textile goods, and wood end products (excluding paper). These goods are staples in the consumer expenditure category of final demand. At the other end of the spectrum are "high technology" commodities such as aircraft, along with their engines and equipment, instrumentation, and electronic equipment and components. Because Canada is a large, sparsely populated country, transport and communications have been and will continue to be vital services; this indicates the importance of the appearance in this set of communication services (including radio and television broadcasting), printed matter such as newspapers and periodicals, and all forms of transport except water transport. And finally there is the class of motor vehicles; the major role of passenger automobiles and motor vehicles in general in the Canadian economy and way of life make it worthwhile to present these commodities in more detail in Table 10. In fact, when the BTEI "with imports" is weighted by the 1966 domestic demand, passenger automobiles are by far the largest single contributor to the overall positive TEC.

TABLE 10

The Energy Costs of Motor Vehicles
(Thousands of BTU's per dollar)

	37200 Passenger Automobiles & Chassis	37300 Trucks, Chassis, Tractors	37400 Buses & Chassis
1. CEEI	59.57	60.07	59.57
Energy Costs with imports			
2. BTEI	73.52	71.30	69.35
3. TEC (2-1)	13.95	11.23	9.78
4. TEC as % of CEEI	23.42%	18.69%	16.42%
Energy Costs, no direct imports			
5. BTEI	66.08	66.32	66.08
6. TEC (5-1)	6.51	6.25	6.51
7. TEC as % of CEEI	10.93%	10.40%	10.93%

Another set of goods for which BTEI exceeds CEEI is the class of primary non-ferrous metals, including nickel, copper, lead and zinc in an unfabricated state. However, given the insignificant flows of these goods to domestic final demand, the energy impact of this set is not great.

The final point worth noting is that the present growth and expected continued expansion of the services sector means a continuing and probably increasing impact of these commodities on the energy trade balance, although this effect will be moderated by the relatively low TEC associated with these goods.

Another of the analytical uses of the balanced trade energy intensity is to calculate the energy impact of various categories of final demand. Table 11 presents just one of many possible breakdowns of demand based on the over 130 categories recognized in the 1966 Input-Output accounts. The calculation is based on the "with import" TEC and a total domestic demand which excludes changes in inventories.

As a general statement, then, investment has the highest trade energy cost, both investment in machinery and equipment, and consumer expenditures on durables and semi-durables, which may be considered a form of investment by the household sector. Since government investment in machinery and equipment has the highest TEC, 16.62 thousand BTU's per dollar, it is the most sensitive category of demand, in the sense that the rate of change of trade energy costs is greatest with respect to changes

TABLE 11

Trade Energy Cost by Category of Demand - 1966

(Thousands of BTU's per dollar)

	(1) % of Total Domestic Demand	(2) TEC	(3) TEC Weighted by (1)
Consumer Expenditures:			
Durables	8.7%	7.46	0.65
Semi-durables	8.0%	4.95	0.40
Non-durables	19.8%	1.77	0.35
Services	22.3%	1.13	0.25
Machinery & Equipment:			
Investment by industry	8.4%	12.07	1.01
Investment by government	0.6%	16.62	0.10
Construction:			
Industry	11.6%	0.18	0.02
Government	4.0%	-1.10	-0.04
Government Current Expenditures	16.7%	1.71	0.29

in this classification. The two high-growth categories of demand, consumer expenditures on services, and government current expenditures, both exhibit a relatively low TEC.

(iii) <u>The Distribution of Trade Energy Costs</u>

Basic to any understanding of the distribution of trade energy costs is the set of closed economy energy intensities; however, given that we are dealing with over six hundred goods, there is perhaps more information than can be easily grasped in printing out a string of numbers. Accordingly, the set of CEEI's is presented in Figure 1 in the form of a frequency diagram, which may be summarized as follows:

(a) there is considerable variation in intensities, roughly one hundred-fold from lowest to highest;

(b) the distribution is roughly trimodal with peaks at about 55, 150, and 220 thousand BTU's per dollar;

(c) nearly 80% of the distinguishable commodities have energy costs less than 100 thousand BTU's per dollar.

Comment (c) points out, and a glance at Figure 1 verifies, that the goods exported, with an energy intensity of 92944 BTU's per dollar (see the note in section 6) are distinctly more energy intensive than the bulk of goods produced in Canada.

Figures 2 and 3 are respectively the frequency diagrams of the "with import" and "no direct import" trade energy cost calculations; both distributions are unimodal, with Figure 2 being more nearly symmetric than Figure 3. The major difference between the two sets of TEC's is the much greater dispersion of

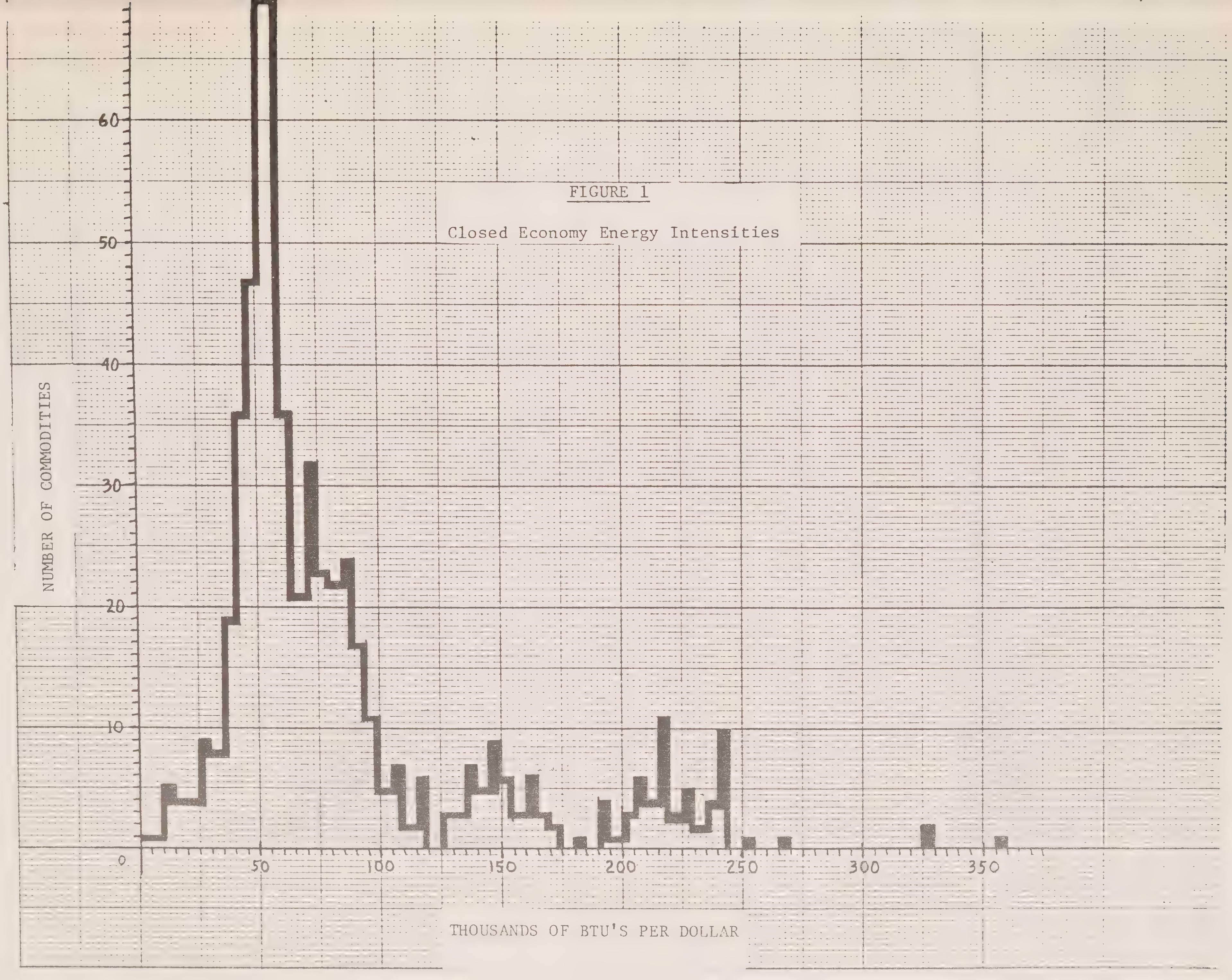


FIGURE 1

Closed Economy Energy Intensities

FIGURE 2

Trade Energy Costs
"With Imports"

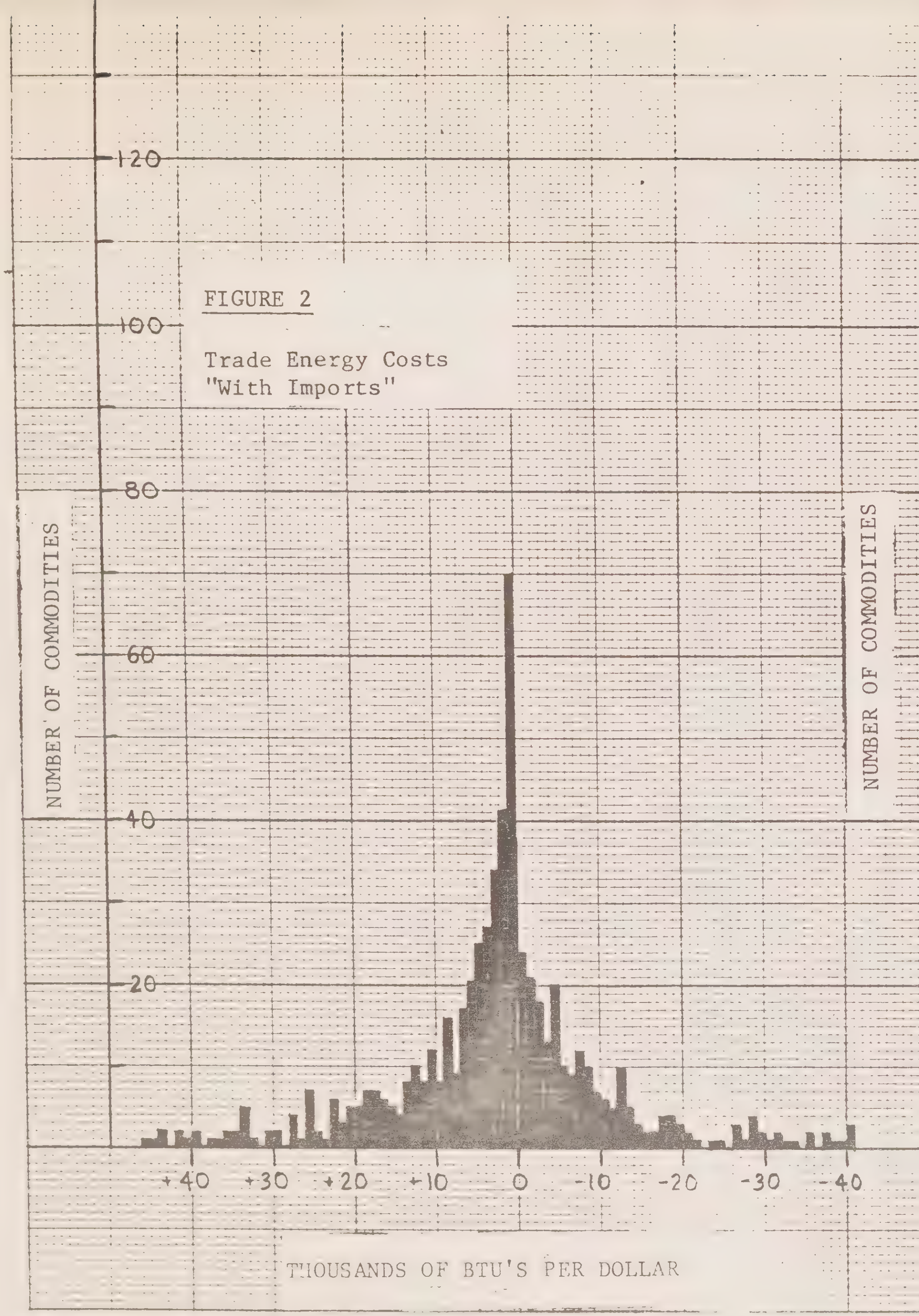
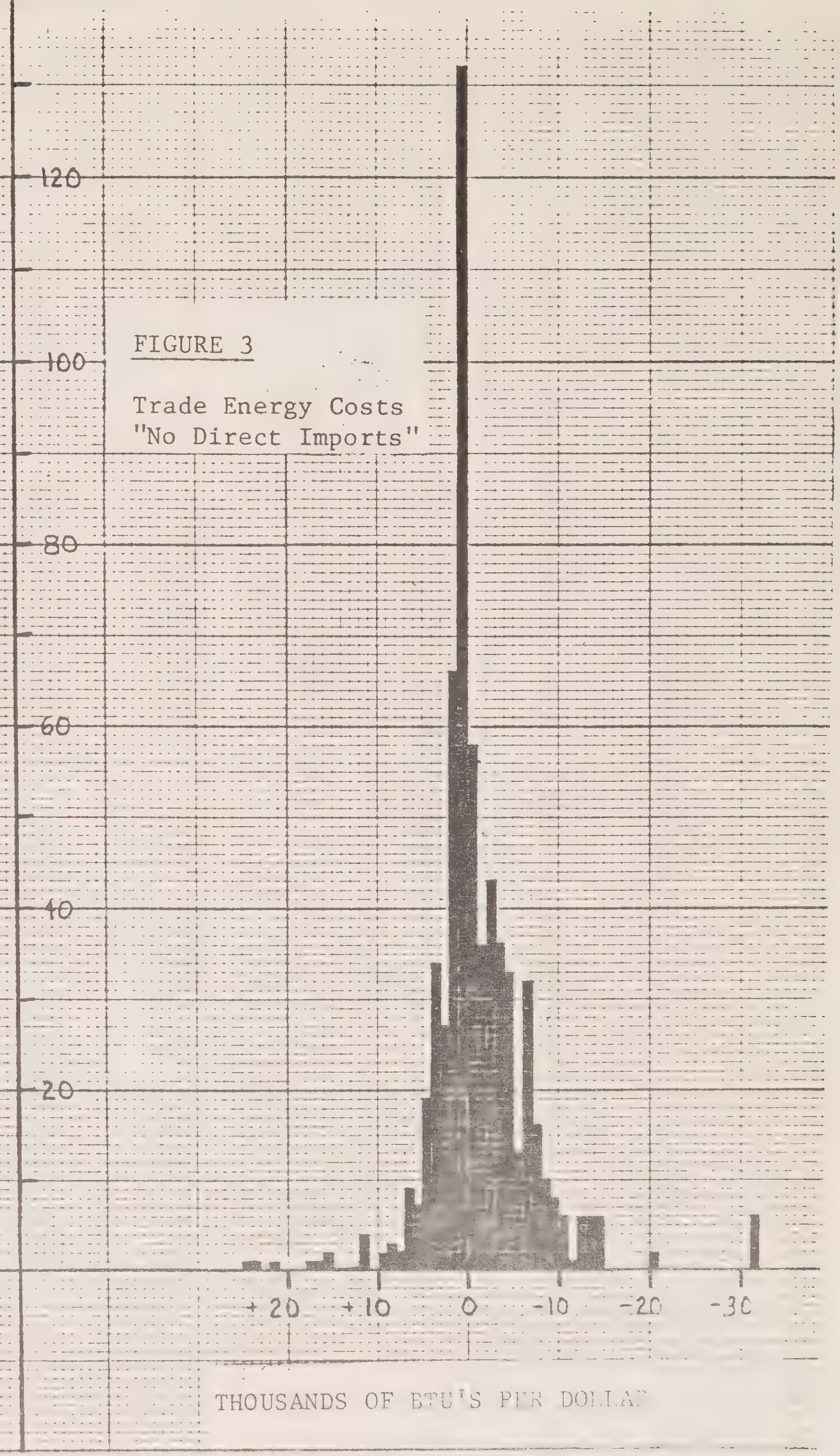


FIGURE 3

Trade Energy Costs
"No Direct Imports"



the "with import" TEC, a phenomenon which will be explained a little later in this section. The two distributions are most similar in the characteristic of having the largest single class of commodities being those with positive trade energy costs between zero and one thousand BTU's per dollar, although this class is more extremely dominant in the "no direct import" case. This would lead one to suspect that, other things being equal, the TEC is highly sensitive to changes in the energy cost per net export dollar (hereafter given the symbol EC/NX$) of the bill of goods exported.

This sensitivity may be examined by supposing that the EC/NX$ of the goods exported is a variable and calculating, for each commodity, the value of this variable for which BTEI would equal CEEI, that is, for which TEC equals 0. Therefore, if ε is the closed economy energy intensity of a given commodity, and γ the variable EC/NX$, recalling formula (1) we have:

$$0 = \text{BTEI} - \text{CEEI} = \varepsilon^{m} + \gamma z - \varepsilon \qquad (2)$$

so that,

$$\gamma = \frac{\varepsilon - \varepsilon^{m}}{z} \qquad (3)$$

As an immediate consequence, given the 1966 EC/NX$ of 92453 BTU's per dollar from Table 7, we have that,

(i) if $\gamma > 92453$, then TEC < 0 (4)

(ii) if $\gamma < 92453$, then TEC > 0, (5)

so that a frequency diagram of the value of γ (hereafter referred to as the <u>break even</u> energy cost per net export dollar of the given commodity) for all commodities would possess the same number of elements less than 92453 as there are goods having positive TEC's in Figures 2 or 3. These frequency diagrams are presented in Figure 4, the "with import" calculation, and Figure 5, the "no direct import" calculation, with class sizes being 5% of 92453 BTU's per dollar.

The number γ associated with each commodity as defined by expression (3) admits of another interpretation. The numerator of expression (3), $\varepsilon-\varepsilon^m$, is just the difference between the CEEI and the energy intensity when imports are allowed (that is, with or without direct imports according to the particular calculation); it will be shown that this difference is equal to the closed economy energy cost of the bill of imported goods necessitated by delivery of one dollar of the commodity in question to final demand. This bill of goods has, by definition, a total value of z, the import intensity, and it therefore follows that γ is the CEEI, that is, the closed economy energy <u>cost</u> normalized to one dollar, of the imports associated with a particular commodity. Expressions (4) and (5) thus clearly state that it is the relation between the CEEI of

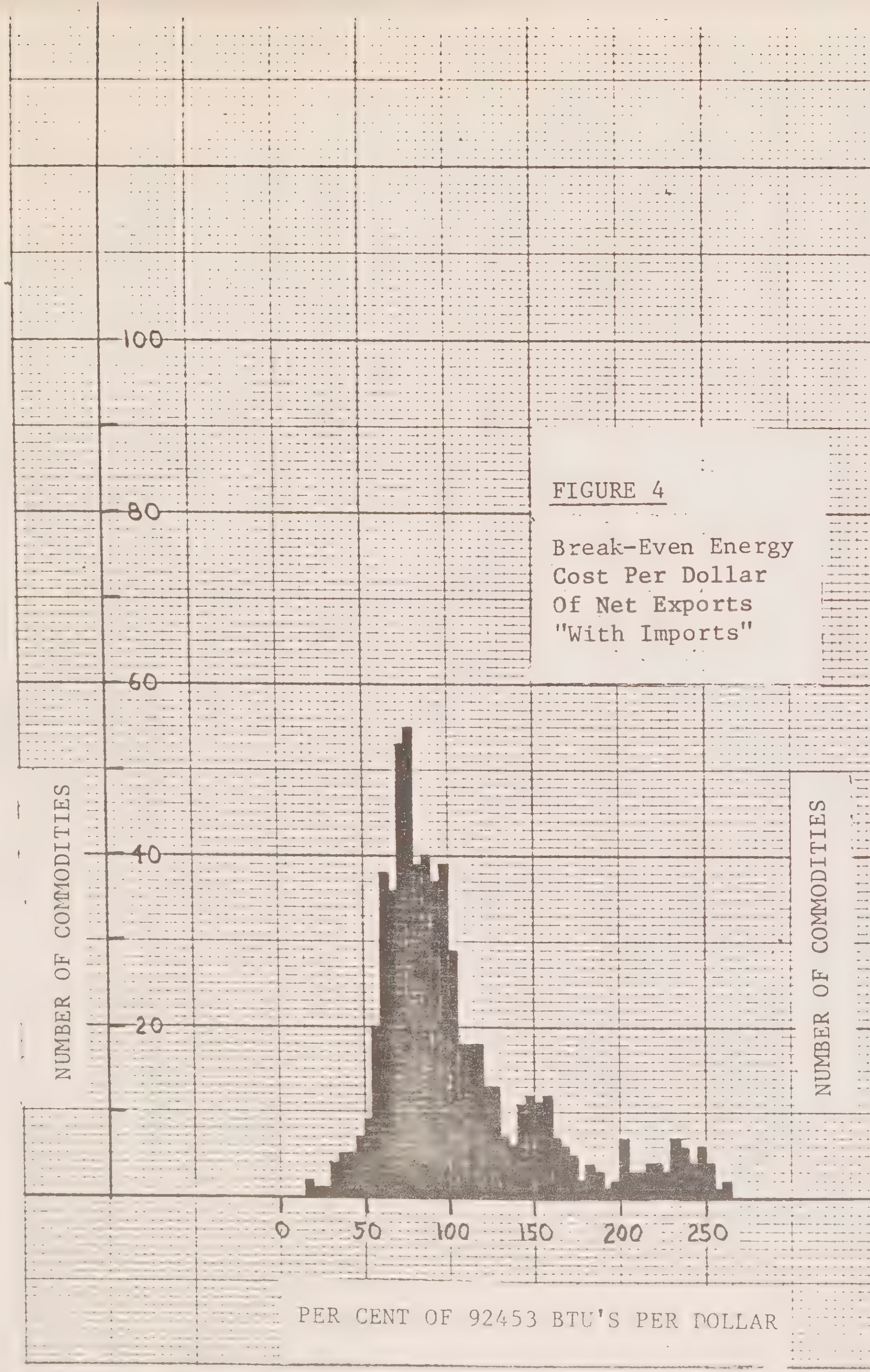


FIGURE 4

Break-Even Energy
Cost Per Dollar
Of Net Exports
"With Imports"

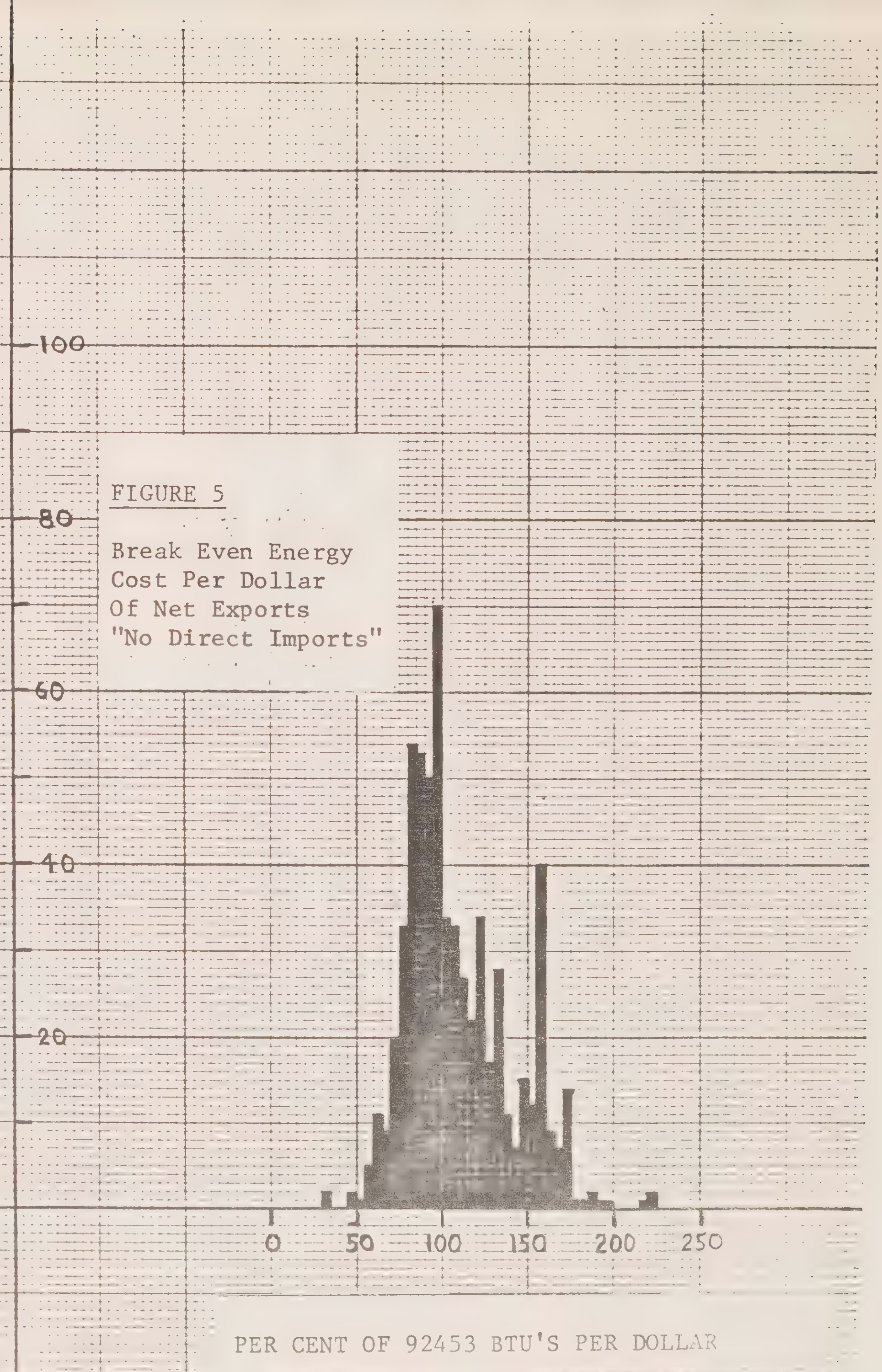


FIGURE 5

Break Even Energy
Cost Per Dollar
Of Net Exports
"No Direct Imports"

the imports needed to produce a commodity and the EC/NX$ of the goods exported which determines the ranking of BTEI and CEEI for that commodity; if the goods imported are less energy intensive per dollar than the exports are per net export dollar, then there is a positive TEC, a higher energy use as a result of balanced external trade than would occur in a closed economy.

Figures 4 and 5 show that this commodity specific import energy intensity exhibits a great deal of variability, although, as in the TEC diagrams, the "with import" calculation is more broadly dispersed.

These diagrams provide a measure of sensitivity as well, for, as a consequence of expressions (4) and (5), it is possible to read Figure 4 (for instance) as saying that an increase of 5% in the EC/NX$ would add another 29 commodities to the class with positive TEC's. The graphs demonstrate a considerable sensitivity of the TEC calculation to variations in the EC/NX$, since the distributions cluster around the figure of 92453 BTU's per dollar. Both calculations are more sensitive to decreases in the EC/NX$ than to increases, this being particularly true of the "no direct import" distribution, whose mode lies in the interval between 0 and -5%, but this sensitivity is perhaps less than the TEC distribution would lead one to expect: (refer to FIGURES 3 and 5) a decrease of 10% in the EC/NX$, which would be a major change in the bill of goods exported, eliminates 120 commodities from the positive TEC class, and yet there are 132

goods with a positive trade energy cost less than 1000 BTU's per dollar.

It should be noted that Figure 4 bears a family resemblance to Figure 1, the distribution of closed economy energy intensities, and this, of course, is not accidental. It turns out that when direct imports are allowed in these calculations, the direct import of the good in question tends to dominate the bill of imported commodities specific to production of this good. For instance, if there existed an "average" commodity whose import share was 0.27, the mean import share of all commodities having non-zero import shares, and whose "no direct import" import intensity was 0.14, the mean of that set of intensities, then it may be shown that this "average" commodity would constitute at least 73% of the total value of the bill of goods appearing as direct and intermediate imports necessary for its production. Thus the break even EC/NX$ of each commodity having a non-zero import share will tend to be close to the CEEI of the commodity, explaining the resemblance of Figures 1 and 4. Moreover, as a consequence of expressions (4) and (5), this means that commodities with a CEEI less than 92453 BTU's per dollar will tend to have positive TEC's by this calculation, and those greater to have negative TEC's, a tendency borne out by Table 8.

The above observation leads to a better understanding of the role of direct imports in the balanced trade energy calculations. For a given commodity it is possible to express the 'with import' trade energy cost (WMTEC) as a function of the

"no direct import" trade energy cost (NDMTEC) and the proportion of each dollar's total demand for the commodity which is met directly by imports, that is, the direct import share (μ_D); letting β equal 92453 BTU's, and ε^m and z be the "no direct import" energy intensity and import intensity respectively of the particular commodity, we have:

$$\text{WMTEC} = \text{NDMTEC} + \mu_D [\beta(1-z) - \varepsilon^m] \qquad (6)$$

From expression (6) we see first that, as noted previously, if μ_D equals zero then the two trade energy costs are equal. We can conclude as well the following:

(i) if the bracketed term equals zero, the two TEC's are equal independent of the direct import share;

(ii) if this term is greater than zero then WMTEC increases when μ_D increases;

(iii) if this term is less than zero then WMTEC decreases when μ_D increases.

In more mathematical language one would say that the bracketed term represents the rate of change of WMTEC with respect to μ_D. The magnitude and sign of this rate of change is dependent on the relation between β and the ratio $\varepsilon^m/(1-z)$; it is clear that, restating points (i) to (iii) above,

(i) if $\frac{\varepsilon^m}{1-z} = \beta$, then the rate of change is zero;

(ii) if $\frac{\varepsilon^m}{1-z} < \beta$, then the rate of change is positive;

(iii) if $\frac{\varepsilon^m}{1-z} > \beta$, the rate of change is negative.

The interest of the ratio $\varepsilon^m/(1-z)$ is that this is in fact the energy cost per net export dollar of the commodity in question. To make this explicit, note that one dollar's worth of the commodity which is to be exported has an energy cost of ε^m, and an associated value of imports of z, so that the net exports are (1-z), and therefore the EC/NX$ is $\varepsilon^m/(1-z)$. The energy cost per net export dollar is thus calculated for an individual commodity in a manner exactly analogous to the way that the figure for β of 92453 BTU's was calculated for the 1966 pattern of exports.

These results can be summarized by the statement that, assuming balanced trade, direct imports of goods whose EC/NX$ exceed 92453 BTU's will decrease the trade energy costs, and direct imports of goods whose EC/NX$ are less than 92453 BTU's will increase the trade energy costs. However, 87% of the commodities fall into the latter class, and, of those in the former, many are goods with no significant flows to domestic final consumption, such as primary non-ferrous metals, basic and fabricated steel, and industrial chemicals. These results also explain the greater dispersion of the WMTEC (Figure 2) as compared with the NDMTEC (Figure 3), for it turns out that most of the goods with a positive NDMTEC also have an EC/NX$ less than 92453 BTU's, and most of those with a negative NDMTEC have an EC/NX$ greater than this figure.

8. Exports and Balanced Trade

As was amply demonstrated in the preceding section, the single most critical parameter in balanced trade energy calculations is the energy cost per net export dollar of the bill of goods exported. The detailed make-up of this bill of goods is therefore of some interest, as is a discussion of the general properties of the EC/NX$.

The last section of this paper introduced the notion of the EC/NX$ of an individual commodity as the "no direct import" energy intensity divided by one minus the import intensity (also with no direct imports and given the symbol NDMII). In discussing a bill of goods we can consider the weight of each commodity to be the proportion of the total value of the bill of goods constituted by that commodity, then calculate the EC/NX$ by taking the weighted sum of the energy intensities and dividing by the weighted sum of the quantities [1-NDMII]. This formulation leads to a pair of simple results. In the first place, the EC/NX$ of a bill of goods is bounded by the EC/NX$ of the individual commodities in it, so that the EC/NX$ of the bill of goods is greater than the minimum EC/NX$ of the commodities involved and less than the maximum. Secondly, given the 1966 exports with an energy cost of β = 92.453 thousand BTU's per net export dollar for example, increasing the proportion in the bill of goods of any commodity with an energy cost greater than β will increase the EC/NX$ of the bill of goods as a whole, and similarly decreasing the proportions will decrease the EC/NX$, both cases assuming the same <u>relative</u> proportion of all other

commodities. Alternatively, selecting a product with an EC/NX$ less than β will have the opposite effect.

These results point out that the important parameters in determining the EC/NX$ of a bill of goods are the energy costs per net export dollar of the particular commodities being exported, as well as, of course, the proportions these commodities make up of the total value of exports. Thus a possible, though perhaps drastic, means of achieving a pattern of exports with an EC/NX$ less than or equal to a specified target figure would be to disallow exports of those goods whose EC/NX$ exceed this figure. In terms of altering the total EC/NX$ by varying the proportions of those commodities whose EC/NX$, for instance, is less than 92.453 thousand BTU's, it is noteworthy that this class of goods is fully 87% of the total number of distinct commodities appearing as exports in 1966.

The energy cost per net export dollar is a function of three sets of parameters: the weight of each commodity in the bill of export goods, and the "no direct import" energy and import intensities of these goods. These parameters therefore determine which commodities will dominate the bill of export goods, and it is possible to select these dominant commodities on this basis, with the results appearing in Table 12. The "% of total energy" figures are arrived at by multiplying the proportion that a given commodity forms of $1 worth of exports by its NDMEI and expressing the result as a percentage of the total NDMEI of exports of 80.064 thousand BTU's per dollar as

TABLE 12

Exports - Dominant Commodities

Energy Figures in Thousands of BTU's Per Dollar (or per net export dollar)

Commodity		% of Value of Exports	NDMEI	% of Total Energy	NDMII	% of Total Imports	EC/NX$
00800	Wheat, unmilled	8.80	51.511	5.66	.064	4.20	55.023
03400	Iron ores & concentrates	2.89	96.885	3.50	.105	2.26	108.218
03600	Gold and alloys, primary	1.20	68.552	1.03	.100	0.90	76.160
04200	Asbestos, unmanfactured	1.55	61.049	1.18	.058	0.68	64.831
07600	Fish products	1.54	71.802	1.38	.140	1.61	83.497
10600	Oilseed, meal & cake	0.20	29.830	0.08	.726	1.11	109.066
21000	Lumber and timber	4.50	38.599	2.17	.073	2.46	41.644
22910	Pulp	4.96	139.954	8.67	.060	2.21	148.834
23000	Newsprint	9.01	153.969	17.32	.075	5.06	166.457
27900	Nickel - primary forms	3.65	84.891	3.87	.152	4.14	100.060
28000	Copper & copper alloys, primary	1.94	83.678	2.03	.155	2.25	99.014
28300	Aluminum & aluminum alloys, primary	2.86	164.415	5.87	.281	6.01	228.742
29000	Aluminum & aluminum alloys, cast	0.57	103.364	0.74	.243	1.04	136.622
35100	Agric. machinery, excl. tractors	1.52	50.481	0.96	.222	2.52	64.907
36000	Machinery - ind. spec.	0.91	47.801	0.55	.178	1.22	58.151
36900	Aircraft engines	0.68	23.897	0.20	.252	1.28	31.926
37000	Specialized aircraft equipment	1.06	26.260	0.35	.243	1.94	34.712
37200	Passenger autos and chassis	4.10	25.160	1.29	.443	13.58	45.141
37300	Trucks, chassis, tractors	1.39	26.162	0.45	.434	4.50	46.253
37900	Motor vehicle engines and parts	1.13	56.060	0.79	.228	1.93	72.659
38100	Motor vehicle access, parts	1.89	48.350	1.14	.288	4.07	67.896
45000	Fertilizers	0.75	117.960	1.11	.143	0.81	137.697
52300	Fertilizer chemicals	0.48	208.300	1.24	.126	0.45	238.318
52400	Synthetic rubber	0.45	224.602	1.27	.124	0.42	256.331
	TOTALS	58.03%		62.85%		65.04%	

presented in Table 7, with a similar procedure employed to arrive at the "% of total imports" figure (recall that exports in 1966 had an overall import intensity of 0.134 import dollars per export dollar). Of the total of over 500 commodities exported by Canada, these 24 goods account for about 60% of the total value, total energy, and total imports associated with the bill of export goods, so that these are indeed the dominant commodities.

As noted earlier, the bill of export goods has a very high EC/NX$, lying in the 87th percentile of the distribution of energy costs per net export dollar of individual commodities. The important structural parameters affecting this value are best explored by considering the difference between the CEEI and the EC/NX$ of the bill of export goods. It may be proved that the EC/NX$ minus the CEEI is equal to the product of two factors; the first of these is the CEEI minus the closed economy energy intensity of the bill of goods imported to produce the exports; the second is an "import intensity factor" consisting of the import intensity of the bill of export goods divided by one minus this figure. Therefore, if the imports necessary for the production of exports have a lower CEEI than the exports themselves, the EC/NX$ will exceed the CEEI for the bill of export goods; the converse holds as well. Therefore, exports in 1966, with an EC/NX$ of 92.453 and a CEEI of 92.944, had intermediate imports necessary for their production which were 3.174 thousand BTU's per dollar more energy intensive (closed economy) than the exports themselves. However, the import

intensity factor attenuates this effect in general since it is greater than 1.0 only for an NDMII exceeding 0.5. Given that the distribution of "no direct import" import intensities has a mean and median lying close to 0.14, ahd that in fact only three commodities have values higher than 0.5, the import intensity factor will tend to be much less than one. For example, an "average" commodity having import intensity of 0.14 would have a factor of 0.163, and the 1966 bill of export goods, with an import intensity of 0.134 (less than average), has a factor of 0.155.

If the 1966 pattern of exports is relatively low in import intensity, it is high in "no direct import" energy intensity; at 80.064 thousand BTU's per dollar , exports are in the 87th percentile of the distribution of NDMEI for individual commodities. Pulp and newsprint alone accounted for 26% of this energy intensity and, referring to Table 12, eleven goods (omitting oilseed) in the classes of raw materials (from wheat to lumber and timber) and semi-finished goods (from pulp to primary aluminum) accounted for 53% of the total energy intensity.

One of the interesting properties of energy intensities as they are defined in this paper, as energy required per <u>dollar</u> of demand rather than per physical unit demanded, is that in most instances a finished, highly manufactured good is less energy intensive than the raw material from which it comes. This may appear contrary to physical principles which state that every

process requires energy, so that there must be "energy added" in each stage of production. In addition to this, however, economics tells us that there must be "value added" as well in each stage of production, this being largely the value of labour. What occurs in measuring an energy intensity as defined in this paper is that in fact what one is measuring is the ratio of two intensities: it is the energy intensity in BTU's per physical unit of the commodity in question, divided by the value added intensity (defined in a manner consistent with the other definitions of "intensity") of that good, in dollars per physical unit. This second intensity is, of course, the price of the commodity.

Physics and economics would indicate that the energy intensity per physical unit and the price are both increasing functions of a variable which could be called "degree of manufacture". However, the nature of a production chain is that the intermediate stages (separation, refining, purification) add much more energy with increasing degree of manufacture than the final stages (tooling, fabrication, assembly, etc.) or the primary stages (extraction). In general the value added increases more or less linearly with degree of manufacture. Under appropriate mathematical assumptions, therefore, the energy intensity per dollar, being the ratio of the above two functions, would increase at the intermediate stages and then decline with increasing manufacture; this decline is such that in many cases the final product is less energy intensive per dollar than the raw material. (For further discussion of this phenomenon refer to footnote 1.)

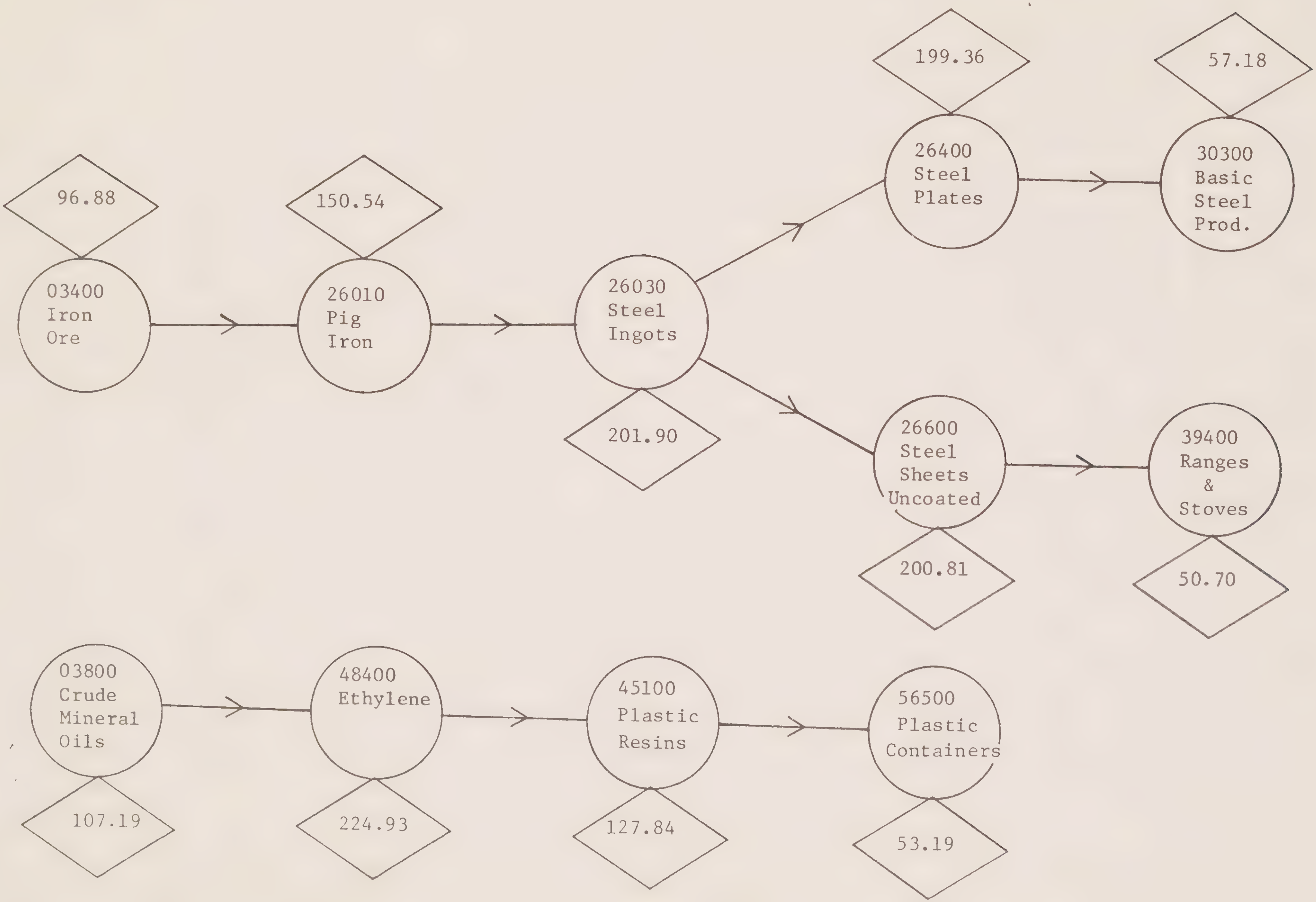
In order to give examples of this phenomenon one would require an Input-Output system in physical units, to show the changes in the individual intensities. Lacking this, it is possible to trace the energy intensity per dollar along production chains and see whether it exhibits the expected "rising and falling" pattern. Diagram 1 shows two production chains for which this is true, and in which each of the final products is less energy intensive than the raw and intermediate goods.

DIAGRAM 1

Simplified Production Chains

For Steel and Plastic Goods

◇ - Indicates "No Direct Import" Energy Intensity in Thousands of BTU's Per Dollar of Demand.



The general statement that finished goods are less energy intensive per dollar than raw or semi-finished goods dictates that, relative to a fixed total value of imports, balancing trade by exporting more highly manufactured commodities would decrease the EC/NX$ relative to its historical value, with corresponding effects on the energy trade balance. This result, by the discussion in the preceding paragraphs, is a corollary to the fact, which has been noted by others, that Canada's role in international trade as a supplier of raw materials means that its exports are not highly labour intensive.

One of the agreeable properties of the EC/NX$ of a bill of goods as defined in this paper is that it is simple to define (in terms of analytical geometry) the set of all possible bills of goods having a fixed energy cost per net export dollar. The interesting question, therefore, having determined some target EC/NX$, involves the choice of criteria for selecting particular bills of goods lying in this set. One possibility is to choose a bill of goods which represents some minimal departure from the historical pattern of exports, the problem then being to define in mathematical terms what is meant by "minimal"; another possibility is to define a variety of programming problems, where some function of the variables, the proportion that each commodity makes up of a dollar's worth of exports, is to be maximized or minimized subject to a fixed target EC/NX$ being achieved.

As an example of this sort of simulation, Table 13 gives the 1966 proportion of total exports constituted by selected commodities, and a calculated proportion arrived at on the following basis:

(1) The target EC/NX$ is 73.963 thousand BTU's per dollar, a 20% reduction from the 1966 level.

(2) The objective in arriving at this target figure is to minimize the sum of the squares of the deviations of individual commodity proportions from the 1966 value.

(3) In performing step (2), exports of goods which were not exported in 1966 are disallowed, although some goods which were exported do go to a zero proportion in arriving at the new pattern.

That this is a more sophisticated procedure than simply increasing the proportions of goods having an EC/NX$ less than 73.963 thousand BTU's is indicated by the fact that all of the dominant exports in Table 12, with the exception of aircraft engines, decrease in the proportion they make up of one dollar's worth of exports.

TABLE 13

Calculated Proportions of Total Exports for a 20% Decrease in Energy

Costs per Net Export Dollar - Selected Commodities

Commodity		% of 1966 Exports	Calculated % of Exports	Difference (2)-(1)
00900	Barley, oats, rye, corn	0.503	0.489	-0.014
02700	Pulpwood	0.254	0.324	0.070
05700	Animal oils, fats & lard	0.111	0.152	0.041
07600	Fish products	1.540	1.279	-0.262
09200	Wheat Flour	0.766	0.715	-0.051
12600	Tobacco, processed, unmfd.	0.322	0.343	0.021
15200	Fabrics, broad-woven, cotton	0.101	0.165	0.064
16500	Fabrics, broad-woven, mix & blends	0.027	0.0	-0.027
21500	Veneer and plywood	0.663	0.666	0.003
23000	Newsprint paper	9.009	7.347	-1.662
25200	Books, pamplets, maps, pictures	0.032	0.120	0.088
26600	Carbon steel sheets, not coated	0.253	0.0	-0.253
30300	Basic steel products	0.022	0.035	0.012
32600	Cutting & forming tools	0.035	0.104	0.069
34000	Forgings of carbon & alloy steel	0.111	0.082	-0.029
35000	Tractors, farm & garden	0.147	0.148	0.001
37000	Specialized aircraft equip.	1.064	1.007	-0.058
39100	Small electric appliances	0.023	0.102	0.080
40200	Electronic equipment comp.	0.216	0.323	0.107
45100	Plastic resins, not shaped	0.179	0.0	-0.179
52300	Fertilizer chemicals	0.477	0.0	-0.477
55300	Aircraft & nautical instr.	0.583	0.631	0.048
55600	Medical and related instr.	0.038	0.142	0.104

9. Mathematical Results

For a complete treatment of the basic mathematics of the I-O model and the associated energy model the reader should see references [1] and [2] . What follows is a description of the essential building blocks and the results relevant to external trade. Given NI industries, NC commodities and NE energy types we have (using the notational conventions of capital letters to represent matrices and small letters column vectors):

B- the input matrix of order NC x NI such that b_{jk} is the proportion the j^{th} good constitutes of the total inputs to the k^{th} industry.

D- the output matrix of order NI x NC such that d_{jk} is the proportion (i.e. market share) industry j produces of the total output of the k^{th} good.

μ- the vector of import coefficients of order NC such that μ_j is the proportion imports of the j^{th} good constitute of the total dometsic supply of good j.

W- the energy matrix of order NE x NI such that w_{jk} is the input of the j^{th} energy type in natural units as a proportion of the total value of production of industry k.

c- an NE-length vector of BTU conversion factors.

$\{e_j\}$- the set of canonical basis vectors for an NC-dimensional Euclidean space, that is, $e_1 = (1,0,0,\ldots,0)$, etc.

Given these structural parameters, the "closed economy" impact table is:

$$(I-DB)^{-1}D \qquad (1)$$

this being a matrix whose j,k^{th} element represents the total production by industry j required to deliver \$1 of the k^{th} good to final demand. Using this

matrix, the vector of industry outputs g is given as a function of the final demand f as,

$$g = (I-DB)^{-1}Df \tag{2}$$

or $$g(f) = (I-DB)^{-1}Df. \tag{2a}$$

The total input BTU's per dollar of industry output is defined by:

$$t' = c'W \tag{3}$$

giving the vector of closed economy energy intensities as

$$\varepsilon' = t'(I-DB)^{-1}D. \tag{4}$$

At this point we assume that there are distinct vectors of import shares for intermediate use (μ^1) and final use (μ^2) and define the following diagonal matrix:

$$\hat{\mu}^1 \text{ s.t. } \quad (\hat{\mu}^1)_{jj} = \mu^1_j$$
$$(\hat{\mu}^1)_{jk} = 0, \quad j \neq k$$

The fixed import share assumption implies that intermediate imports m^1 may be defined as the following function of final demand:

$$m^1 = \hat{\mu}^1 Bg(f) \tag{5}$$

and the direct imports as,

$$m^2 = \hat{\mu}^2 f \tag{6}$$

giving total imports $m = m^1 + m^2$.

The vector of domestic commodity production q as a function of final demand f is the sum of intermediate and final use in the closed economy case,

$$q(f) = Bg(f) + f. \tag{7}$$

If intermediate imports are allowed, this becomes

$$q(f) = Bg(f) - \hat{\mu}^1 Bg(f) + f \quad \text{from (5)} \tag{8}$$

$$= (I-\hat{\mu}^1)\, Bg(f) + f.$$

Assuming the same market shares apply to intermediate and final use, industry outputs are obtained by distributing commodity production according to these shares, i.e.

$$g(f) = Dq(f)$$

$$= D(I-\hat{\mu}^1)Bg(f) + Df$$

$$\text{or} \qquad (I-D(I-\hat{\mu}^1)B)g(f) = Df$$

giving as solution

$$g(f) = (I-D(I-\hat{\mu}^1)B)^{-1}Df. \tag{9}$$

(The condition $0 \le \mu^1_j < 1$ for all $j = 1,\ldots,NC$, together with the existence of the inverse in (1), are sufficient to guarantee the existence of this inverse.) Therefore the "no direct import" energy intensity may be written as

$$\varepsilon^{m'1} = t'(I-D(I-\hat{\mu}^1)B)^{-1}D \tag{10}$$

It is clear that the "with import" calculation (that is with both intermediate and direct imports) is simply

$$\varepsilon^{m_1 m_2'} = \varepsilon^{m_1'}(I-\hat{\mu}^2).$$

$$= t'(I-D(I-\hat{\mu}^1)B)^{-1}D(I-\hat{\mu}^2)\,. \tag{11}$$

To simplify the notation, any energy intensity involving imports will be written as $\varepsilon^{m'}$, and if direct imports are explicitly allowed this can be written as $\varepsilon^{m'}(I-\hat{\mu}^2)$.

Introducing a vector of import shares in physical terms for the energy commodities, μ^* of length NE, the vector of BTU's of domestic energy inputs as a proportion of the value of industry output is

$$s' = c'(I-\hat{\mu}^*)W. \tag{12}$$

The domestic energy intensities may thus be written as

$$\varepsilon^{d'} = s'(I-D(I-\hat{\mu}^1)B)^{-1}D \tag{13}$$

and, again, if direct imports are allowed this is just $\varepsilon^{d'}(I-\hat{\mu}^2)$.

Before developing the formulation of the balanced trade energy intensity it is first necessary to define the import intensity. Assuming for the moment that there are no direct imports we have,

$$m^1 = \hat{\mu}^1 Bg(f)$$

$$= \hat{\mu}^1 B(I-D(I-\hat{\mu}^1)B)^{-1}Df \qquad \text{from (5) and (9).}$$

Thus total imports as a function of demand, M(f), is given by

$$M(f) = \mu^{1'} B(I-D(I-\hat{\mu}^1)B)^{-1}Df \tag{14}$$

so that the vector,

$$z' = \mu^{1'} B(I-D(I-\hat{\mu}^1)B)^{-1}D \tag{15}$$

has as its j^{th} element the total value of imports required as intermediate inputs in producing \$1 of the j^{th} good for final demand. When direct imports are allowed the vector of import intensities becomes $z'(I-\hat{\mu}^2) + \mu^2$.

For the pattern vector of exports, r (defined such that r_j is the fixed proportion of \$1 of exports constituted by the j^{th} good), the objective in balanced trade calculations is to find, for any final demand f, the total value of exports X which will balance trade. The value of imports associated with total demand (i.e. domestic demand plus exports) is,

$$M = z'f + Xz'r. \tag{16}$$

Balancing trade we have,

$$M = X = z'f + Xz'r \tag{17}$$

so that

$$X = \frac{z'f}{1-z'r}\ . \tag{18}$$

For a final demand of \$1 of the j^{th} good this becomes

$$X = \frac{z_j}{1-z'r} \qquad (19)$$

The export vector has an energy intensity of $\varepsilon^{m'}r$; the balanced trade energy intensity of the j^{th} good is defined as the sum of the (in this case) "no direct import" energy intensity of this commodity plus the energy cost of those exports required to balance trade, that is

$$\begin{aligned}\varepsilon_j^b &= \varepsilon_j^m + [\varepsilon^{m'}r]X \\ &= \varepsilon_j^m + [\varepsilon^{m'}r]\frac{z_j}{1-z'r} \qquad \text{from (19)} \\ &= \varepsilon_j^m + \frac{\varepsilon^{m'}r}{1-z'r}z_j. \qquad (20)\end{aligned}$$

This formula is more complicated in the presence of direct energy commodity trade; in the first place, the appropriate vector of energy intensities to use is ε^d; secondly, the direct exports of energy have a total BTU content of $p'\hat{c}r^*$, where r^* is the NE length vector of proportions of \$1 of exports constituted by the energy commodities and p the NE length vector of energy commodity prices. The balanced trade energy intensity now becomes

$$\varepsilon_j^b = \varepsilon_j^d + \frac{\varepsilon^{d'}r + p'\hat{c}r^*}{1-z'r}z_j. \qquad (21)$$

Referring to statement (20), note that the expression $(\varepsilon^{m'}r)/(1-z'r)$ is in fact the energy cost per net export dollar of the vector of exports, since the term in the denominator is the value of exports less the value of imports needed in their production. Assigning this parameter the symbol β and substituting from expressions (10) and (15) we may write,

$$\varepsilon^{b'} = [t' + \beta\mu^{1'}B](I-D(I-\hat{\mu}^1)B)^{-1}D. \qquad (22)$$

Summarizing the mathematical expressions of the four definitions of energy intensity used in this paper, we have:

1. The closed economy energy intensity,

$$\varepsilon' = t'(I-DB)^{-1}D.$$

2. The "with import" energy intensity,

$$\varepsilon^{m'} = t'(I-D\ (I-\hat{\mu}^{1})\ B)^{-1}D.$$

3. The domestic energy intensity,

$$\varepsilon^{d'} = s'(I-D(I-\hat{\mu}^{1})B)^{-1}D$$

4. The balanced trade energy intensity,

$$\varepsilon^{b'} = [t' + \beta\mu^{1'}B](I-D\ (I-\hat{\mu}^{1})\ B)^{-1}D.$$

Before proceeding with the analysis, a couple of general results are required.

<u>Proposition 1</u> The matrices DB and BD possess the same non-zero eigenvalues.

<u>proof</u>: Suppose $\lambda \neq 0$ is any eigenvalue of DB and x the associated eigenvector.

$\therefore$ $DBx = \lambda x$

implying $BDBx = B\lambda x$

implying $BD(Bx) = \lambda(Bx)$.

$\therefore$ λ is an eigenvalue of BD.

Now suppose that $\gamma \neq 0$ is any eigenvalue of BD and y the associated eigenvector.

$\therefore$ $BDy = \gamma y$

implying $DBDy = D\gamma y$

implying $DB(Dy) = \gamma(Dy)$.

$\therefore$ γ is an eigenvalue of DB. Q.E.D.

<u>Corollary 1</u> I-XY is invertible iff I-YX is invertible for any matrices X and Y of compatible dimension.

The proof of Corollary 1 is obvious.

<u>Proposition 2</u> Given that one of $(I-XY)^{-1}$ or $(I-YX)^{-1}$ exists,

then $(I-XY)^{-1}X = X(I-YX)^{-1}$.

<u>proof</u>: By Corollary 1, both sides of the expression are defined. Now,

$$
\begin{aligned}
(I-XY)^{-1}X &= (I-XY)^{-1}\ X\ (I-YX)\ (I-YX)^{-1} \\
&= (I-XY)^{-1}(X-XYX)\ (I-YX)^{-1} \\
&= (I-XY)^{-1}(I-XY)\ X\ (I-YX)^{-1} \\
&= X(I-YX)^{-1}
\end{aligned}
$$

Q.E.D.

We now turn to a more specific result.

<u>Proposition 3</u> Given $m = \hat{\mu}^1Bg + \hat{\mu}^2f$, where $g = (I-D(I-\hat{\mu}^1)B)^{-1}D[(I-\hat{\mu}^2)f + x]$,

then for any f and x,

$$(I-DB)^{-1}D(f + x - m) = (I-D(I-\hat{\mu}^1)B)^{-1}D[(I-\hat{\mu}^2)f + x].$$

<u>proof</u>: Proposition 2 and Corollary 1 are used repeatedly:

$$
\begin{aligned}
(I-DB)^{-1}D(f + x - m) &= (I-DB)^{-1}D\{f + x - \hat{\mu}^1B(I-D(I-\hat{\mu}^1)B)^{-1}D[(I-\hat{\mu}^2)f + x] - \hat{\mu}^2f\} \\
&= (I-DB)^{-1}D[I - \hat{\mu}^1B(I-D(I-\hat{\mu}^1)B)^{-1}D]\ [(I-\hat{\mu}^2)f + x] \\
&= D(I-BD)^{-1}\ [I - \hat{\mu}^1BD\ (I-(I-\hat{\mu}^1)BD)^{-1}]\ [(I-\hat{\mu}^2)f + x] \\
&= D(I-BD)^{-1}\ [I-(I-\hat{\mu}^1)BD - \hat{\mu}^1BD]\ (I - (I-\hat{\mu}^1)BD^{-1}\ [(I-\hat{\mu}^2)f + x] \\
&= D(I-BD)^{-1}\ [I-BD]\ (I - (I-\hat{\mu}^1)BD)^{-1}\ [(I-\hat{\mu}^2)f + x] \\
&= (I-D(I-\hat{\mu}^1)B)^{-1}D[(I-\hat{\mu}^2)f + x]
\end{aligned}
$$

Q.E.D.

Proposition 3 sheds light on the difference between energy cost simulations of open and closed economies as presented in the body of the paper. Ignoring direct flows of energy for the sake of simplicity and letting f be domestic demand, x be exports, and m be imports, the energy costs of a closed economy are

$$CE = \varepsilon'f$$

For an open economy the energy costs are

$$OE = \varepsilon^{m\prime}(I-\hat{\mu}^2)f + \varepsilon^{m\prime}x$$

$$= \varepsilon^{m\prime}[(I-\hat{\mu}^2)f + x]$$

$$= \varepsilon'[f + x - m] \quad \text{by Proposition 3.}$$

$$\therefore\ OE - CE = \varepsilon'x - \varepsilon'm. \tag{23}$$

Expression (23) states that the determining factor in the ranking of these simulated total energy costs is the difference in closed economy energy costs of exports as compared with imports. This latter difference is a function of the respective total values of imports and exports and the difference in closed economy energy intensity of the two vectors. Trade energy costs can now be explained as well. The vector of trade energy costs is given by

$$d = \varepsilon^m + \beta z - \varepsilon \tag{24}$$

Thus $$d_j = \varepsilon_j^m + \beta z_j - \varepsilon_j$$

$$= z_j \left(\beta - \frac{\varepsilon_j - \varepsilon_j^m}{z_j} \right) \tag{25}$$

Denoting the vector of imports needed to deliver \$1 of the j^{th} good to final demand as $m(e_j)$, we observe that this vector by definition has a total of z_j. By Proposition 3 we therefore have:

$$\varepsilon_j^m = \varepsilon'(e_j - m(e_j)).$$

$$= \varepsilon_j - \varepsilon'm(e_j).$$

$$\therefore\ \frac{\varepsilon_j - \varepsilon_j^m}{z_j} = \frac{\varepsilon'm(e_j)}{z_j} = \varepsilon'\overline{m(e_j)} \tag{26}$$

where the bar indicates a vector normalized to sum to one.

$$\therefore\ d_j = z_j\ [\beta - \varepsilon'\overline{m(e_j)}]. \tag{27}$$

Balanced trade calculations therefore lead to the result that if the vector of imports necessary for the production of a commodity has a greater (lower)

closed economy energy intensity than the vector of exports has energy costs per dollar of net exports there is lower (greater) energy use in producing the commodity in question than would occur in the absence of trade.

Expression (24) assumes no direct imports - to explore the effects of direct imports on trade energy costs we re-write this as

$$d^1_j = \varepsilon^m_j + \beta z_j - \varepsilon_j.$$

With direct imports allowed this becomes

$$\begin{aligned} d^2_j &= \varepsilon^m_j (1 - \mu^2_j) + \beta[z_j (1 - \mu^2_j) + \mu^2_j] - \varepsilon_j \\ &= (\varepsilon^m_j + \beta z_j - \varepsilon_j) + \beta\mu^2_j - \beta z_j \mu^2_j - \varepsilon^m_j \mu^2_j \\ &= d^1_j + \mu^2_j \; [\beta(1 - z_j) - \varepsilon^m_j] \\ &= d^1_j + \mu^2_j (1 - z_j) \; [\beta - \frac{\varepsilon^m_j}{1 - z_j}] \end{aligned} \tag{28}$$

Therefore, if the commodity has an energy cost per net export dollar $\varepsilon^m_j/(1-z_j)$ which is greater (less) than that of the total vector of exports, the trade energy cost is less (greater) than that in the case of no direct imports (assuming the commodity has a non-zero import coefficient).

Turning now to a more detailed look at the energy cost per net export dollar of a vector of exports, we have the following basic result:

<u>Proposition 4</u> For any set $S \subset \{1,2,3,\ldots, NC\}$, $S \neq \emptyset$, any non-negative vector w such that $w_k > 0$ for all $k \varepsilon S$, and for $u_j > 0$, $v_j > 0$ for all $j = 1,\ldots, NC$, this inequality holds:

$$\min_{j \varepsilon S} \frac{u_j}{v_j} \leq \frac{\sum_{k \varepsilon S} w_k u_k}{\sum_{k \varepsilon S} w_k v_k} \leq \max_{j \varepsilon S} \frac{u_j}{v_j}$$

proof: without loss of generality, assume $\alpha = \frac{u_1}{v_1} = \max_{j\varepsilon S} \frac{u_j}{v_j}$, $w_1 > 0$, and

$S' = S - \{1\}$

Now

$$\frac{u_j}{v_j} > \alpha \text{ for all } j\varepsilon S'$$

$\therefore$

$$\frac{w_j u_j}{w_j v_j} > \alpha \qquad j\varepsilon S'$$

implying

$$w_j u_j > \alpha\, w_j v_j \qquad j\varepsilon S'$$

implying

$$\sum_{j\varepsilon S'} w_j u_j > \alpha \sum_{j\varepsilon S'} w_j v_j$$

implying

$$\frac{\sum_{j\varepsilon S'} w_j u_j}{\sum_{j\varepsilon S'} w_h v_j} > \alpha$$

Thus

$$\frac{\sum_{j\varepsilon S'} w_j u_j}{\sum_{j\varepsilon S'} w_j v_j} > \frac{w_1 u_1}{w_1 v_1}$$

This expression has the form

$$\frac{c}{d} > \frac{a}{b}\ ,\ a,b,c,d > 0$$

which implies

$$bc - ad > 0$$

implying

$$\frac{ab + bc - ad - ab}{b^2 + bd} > 0$$

implying

$$\frac{a + c}{b + d} - \frac{a}{b} > 0$$

implying

$$\frac{a + c}{b + d} > \frac{a}{b}\ .$$

$$\therefore \quad \frac{\sum_{j\varepsilon S} w_j u_j}{\sum_{j\varepsilon S} w_j v_j} \geq \frac{u_1}{v_1}$$

where equality can only hold if $S = \{1\}$. This argument is clearly symmetric with respect to the direction of the inequality.

Q.E.D.

As a particular case of this we see that the energy cost per net export dollar of a (non-negative) vector of goods is bounded by the energy cost per net export dollar of the individual goods making up the non-zero elements of the vector. For a given vector of exports x, the energy cost per net export dollar is

$$\gamma(x) = \frac{\sum_1^{NC} x_j \varepsilon_j^m}{\sum_1^{NC} x_j (1-z_j)} \tag{29}$$

If an additional p dollars of, for instance, commodity 1 is exported, the proportion of commodity 1 is increased but all other relative proportions x_j/x_k, $j,k \neq 1$, remain the same. The energy cost per net export dollar is now

$$\gamma(x^P) = \frac{(x_1 + p)\varepsilon_1^m + \sum_2^{NC} x_j \varepsilon_j^m}{(x_1 + p)(1 - z_j) + \sum_2^{NC} x_j (1-z_j)}$$

$$= \frac{p\varepsilon_1^m + \sum_1^{NC} x_j \varepsilon_j^m}{p(1-z_1) + \sum_1^{NC} x_j (1-z_j)} \tag{30}$$

By the argument employed in the proof of Proposition 4, it follows that if good 1 has a greater (lesser) energy cost per net export dollar than the original vector , the new vector will have a greater (lesser) energy cost per net

export dollar.

To explain the relationship between the closed economy energy intensity of a vector of exports x, or, for the sake of simplicity, this same vector normalized to sum to one, r, and the energy cost per net export dollar of this vector, consider:

$$\gamma(r) - \varepsilon' r = \frac{\varepsilon^{m'} r}{1-z' r} - \varepsilon' r$$

$$= \frac{\varepsilon' r - \varepsilon' m(r)}{1 - z' r} - \varepsilon' r$$

$$= \frac{\varepsilon' r - \varepsilon' m(r) - \varepsilon' r + (\varepsilon' r)(z' r)}{1 - z' r}$$

$$= \frac{(\varepsilon' r)(z' r) - \varepsilon' m(r)}{1 - z' r}$$

$$= \frac{z' r}{1-z' r} \; [\varepsilon' r - \varepsilon' \overline{m(r)}] \; . \tag{31}$$

Thus, as one consequence among others pertaining to this expression, if the imports needed to produce exports have the same closed economy energy intensity as the exports themselves, then the energy cost per net export dollar of the vector of exports is equal to its closed economy energy intensity. Suppose now that we wish to characterize the set of non-negative vectors having a fixed energy cost per net export dollar α.

$$\text{Therefore} \quad \frac{\varepsilon^{m'} x}{(i-z)' x} = \alpha, \text{ where } i = (1,1,1,\ldots,1)$$

$$\text{or} \quad [\varepsilon^{m} - \alpha(i-z)]' x = 0. \tag{32}$$

This is the equation of a hyperplane through the origin - it will be <u>feasible</u> if it intersects the positive orthant at points other than the origin, so that

there are non-negative vectors x satisfying (32).

Feasibility is equivalent to the vector $\varepsilon^m - \alpha(i - z)$ having both positive and negative elements. Proposition 4 shows that feasibility is guaranteed if α lies between the limits set by the lowest and highest energy costs per net export dollar of individual goods.

APPENDIX 1

BTU CONVERSION FACTORS[1]

Energy Type

1.	Crude Oil	-	5.803	million BTUS/barrel of 35 I.G.	
2.	Coal[2]	-	23.830	"	/ton
3.	Natural Gas	-	1.025	"	/thousand cubic ft.
4.	Electricity	-	3.412	"	/thousand kwh.
5.	Coke	-	24.800	"	/ton
6.	Gasoline	-	0.149	"	/Imp. gallon
7.	Fuel Oil[2]	-	0.170	"	/Imp. gallon
8.	L.P.G.	-	0.117	"	/Imp. gallon
9.	Still gas	-	0.180	"	/Imp. gallon

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1. These factors are based on standards and practices set out in the following document:

   Fuel Conversion Factors, Fuels and Mining Practice Division Internal Report FMP 62/28, Dept. of Mines and Technical Surveys, Mines Branch, March 1962.

2. Coal and fuel oil conversions are weighted averages (weighted according to 1966 domestic supply) of those of the different grades of these fuels.

Footnotes

1. A more general and complete analysis of production chains is contained in a joint Dept. of Energy Mines and Resources - Statistics Canada paper, still under preparation, to be titled Mineral Commodity Production in Canada at Various Levels of Fabrication: An Input-Output Analysis of Economic Factors.

References

1. Users' Guide to Statistics Canada Structural Economic Models, Structural Analysis Division, Statistics Canada, Nov. 1974 (revised Feb. 1976).

2. Hamilton, K.E., and McInnis, B.C., Gross Energy Requirements for the Production of Goods - An I/O Baseline, Structural Analysis Division Working Paper 75-06-01, Statistics Canada.

3. Hamilton, K.E., and McInnis, B.C., Economic Environmental Modelling, Structural Analysis Division Working Paper 74-10-25, Statistics Canada.